ISET MATH II Term Midterm Exam **Name . . . .**

Answers without work or justification will not receive credit.

Problem 1. (1×8 pt) Let $C = \begin{pmatrix} 3 & 7 & 10 & 2 \\ 9 & 5 & 1 & 3 \\ 0 & 2 & 4 & 6 \end{pmatrix}$ and $D = \begin{pmatrix} 8 & 3 \\ 1 & 5 \\ 2 & 0 \\ k & 11 \end{pmatrix}$.

Suppose the a_{11} entry of $C \cdot D$ is 51.

Find each of following values. If the value does not exist write "DNE".

		Answer
(a)	*The* a_{21} *entry of* $C \cdot D$	
(b)	*The* a_{43} *entry of* $C \cdot D$	
(c)	*The value of* k	
(d)	*The* a_{23} *entry of* $(D \cdot C)^T$	
(e)	*The* a_{23} *entry of* $(C \cdot D)^T$	
(f)	*The size of the matrix product* $C \cdot C^T$	
(g)	*The* a_{21} *entry of* $D^T \cdot D$	
(h)	*The* a_{12} *entry of* $D^T \cdot D$	

Solution

Problem 2. In a two-industry economy, it is known that industry I uses 50 cents of its own product and 3 dollar's of commodity II to produce a 5 dollar's worth of commodity I; industry II uses non of its own product but uses 50 cents of commodity I in producing a dollar's worth of commodity II.

(a) Write the input-output matrix and the Leontief matrix of this economy.

(b) If the economy produces \$ 2000 of commodity I and \$ 1000 of commodity II, how much of this production is internally consumed by the economy?

(c) If the economy consumes internally \$ 4000 of commodity I and \$ 3000 of commodity II, how much of external demand can be fulfilled in this case?

(d) Suppose the external demands are \$ 3000 of commodity I and \$ 6000 of commodity II. Find total production which fulfils this demand.

Answer

(a)	
(b)	
(c)	
(d)	

Solution

Problem 3. (From the exam of University of Pennsylvania) Let A and B be square matrices with $AB = 0$. Give a proof or counterexample for each of the following.

a) $BA = 0$.

b) Either $A = 0$ or $B = 0$ (or both).

c) If $det(A) = -3$, then $B = 0$.

d) If B is invertible then $A = 0$.

e) There is a vector $v \neq 0$ such that $BAv = 0$.

Answer

(a)	
(b)	
(c)	
(d)	
(e)	

Solution

Problem 4. (From the exam of University of Pennsylvania) Consider the system of equations

$$\begin{cases} x + y - z &= a \\ x - y + 2z &= b \end{cases}$$

a) Find the general solution of the homogeneous system.

b) A particular solution of the non-homogeneous system when $a = 1$ and $b = 2$ is $x = 1,\ y = 1,\ z = 1$. Find the general solution of the given system in this case.

c) Find the solution of the non-homogeneous system when $a = -1$ and $b = -2$.

d) Find the solution of the non-homogeneous system when $a = 3$ and $b = 6$.

[Remark: After you have done part (a), it is possible immediately to write the solutions to the remaining parts.]

Answer

(a)	
(b)	
(c)	
(d)	

Solution

Problem 5.

(a) Show that if M is a 2×2 Markov matrix, so is M^2.

(b) Fill in the matrix $A = \begin{pmatrix} \frac{1}{2} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ so that A is a positive Markov matrix with the steady vector $v = \begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix}$.

(c) Find a steady vector of A^2.

Answer

(a)	
(b)	
(c)	

Solution

ADDITIONAL PAPER

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