ISET MATH I Term 2013 Midterm Exam

Problem 1. Let $f(x) = 2x^6 + 3x^5$, find
(a) domain and range;
(b) x and y intercepts;
(c) increasing and decreasing intervals;
(d) intervals of concavity up, concavity down and inflection points;
(e) local minimums and maximums;
(f) global minimums and maximums;
(g) min and max on the interval $[-1, 1]$;
(h) sketch the graph.

Answer

(a)	$Domain \;\; x \in \;(\qquad\quad , \qquad\quad)$ $Range \;\; y \in$
(b)	$x - interc. \;\; x =$ $y - interc.$
(c)	$Increasing \;\; x \in (\qquad\quad , \qquad\quad)$ $Decreasing \;\; y \in$
(d)	$conc.\, down \;\; x \in$ $conc.up$ $inflection$
(e)	$loc.\, min \;\; x_{min} =$ $loc.\, max \;\; x_{max} =$
(f)	$glob.\, min$ $glob.\, max$
(g)	$min.\, x_{min} = \qquad\quad , \qquad y_{min} =$ $max.\, x_{max} = \qquad\quad y_{max} =$
(h)	

Solution

Problem 2. Let $f(x) = \frac{16(5-x)}{x^2-16}$. Find:
(a) Domain and range;
(b) All asymptotes;
(c) All local minimums and maximums;
(d) All intercepts
(e) Sketch the graph of f.
Give examples of rational functions satisfying the following conditions:
(f) Vertical asymptotes at $x = 1$ and $x = -1$ and oblique asymptote is $y = -x$.
(g) Vertical asymptote at $x = 1$ and and oblique asymptote is $y = -x$.
(h) No vertical asymptotes and oblique asymptote is $y = -x$.

Answer

(a)	$Domain$ $Range$
(b)	$Vertical$ $Horizontal$ $Oblique$
(c)	$Local\ min$ $Local\ max$
(d)	$y - interc.$ $x - interc.$
(e)	
(f)	
(g)	
(h)	

Solution

Problem 3. The cost function of a firm is given by $C(x) = x^2 + 16$.
(a) Find the value x_0 which minimizes the average cost $AC(x)$.
(b) Assume that the firm is in perfectly competitive situation and it receives for its output a constant price $p(x) = 20$. Calculate: (b_1) the optimal output point, that is the value x^* which maximizes the profit; (b_2) the maximal profit; (b_3) brake even points; (b_4) sketch the graphs of $AC,\ MC,\ MR$ and indicate all intersection points.
(c) Assume now that the firm is in pure monopolistic situation and the price function is given by $p(x) = 40 - x$. Calculate: (c_1) the optimal output point; (c_2) the maximal profit; (c_3) brake even points; (c_4) sketch the graphs of $AC,\ MC,\ MR$ and indicate all intersection points.

Answer

(a)	$x_0 =$	
(b)	$b_1 : x^* =$	$b_2 :$
	$b_3 :$	$b_4 :$
(c)	$c_1 :\ 10$	$c_2 :$
	$c_3 :$	$c_4 :$

Solution

Problem 4. Let $f(x) = \begin{cases} x^2 & x \leq 1 \\ -x^2 + ax + b & x > 1. \end{cases}$

(a) Indicate values of a and b for which $f(x)$ is a continuous but not C^1 function. Plot the graph.

(b) Find the values of a and b for which $f(x)$ is a C^1 function. Plot the graph.

(c) Is the obtained function C^2? Justify your answer.

Answer

(a)	*say* $a =$ $b =$
(b)	$a =$ $b =$
(c)	

Solution

Problem 5. The demand function is given by $x(p) = 120 - 20p$.

(a) Find the elasticity.

(b) At what price is the elasticity equal to -1?

(c) Find the price interval where the demand is elastic?

(d) Find the price interval where the demand is inelastic?

(e) At price $p = 2$ will a small increase in price cause the total revenue to increase or decrease?

(f) At price $p = 4$ will a small increase in price cause the total revenue to increase or decrease?

(g) Calculate percent of change of demand if the price $p_0 = 2$ increases by 10%. Give the answer: (g_1) first by direct calculation, (g_2) then using elasticity.

(h) Calculate percent of change of demand if the price $p_0 = 4$ increases by 10%. Give the answer: (h_1) first by direct calculation, (h_2) then using elasticity.

Answer

(a)	$\epsilon(p) =$
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	(g_1) : (g_2) :
(h)	(h_1) : (h_2) :

Solution

ADDITIONAL PAPER

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