

## Integrals

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#### Antiderivative = Indefinite integral

**Definition.** Indefinite integral  $\int f(x) dx$  is a function  $F(x)$  such that  $F'(x) = f(x)$ .

#### Examples

$$\int 2x dx = x^2 + C, \int 3x^2 dx = x^3 + C, \int e^x dx = e^x + C, \int 1/x dx = \ln x + C.$$

#### Indefinite integral formulas

$$1. \int k dx = kx + C; \quad 2. \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1); \quad 3. \int x^{-1} dx = \ln x + C;$$

$$4. \int k f(x) dx = k \int f(x) dx; \quad 5. \int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx;$$

$$6. \int [u(x)]^n u'(x) dx = \frac{[u(x)]^{n+1}}{n+1} + C \quad (n \neq -1); \quad 7. \int [u(x)]^{-1} u'(x) dx = \ln u(x) + C;$$

$$8. \int e^{u(x)} u'(x) dx = e^{u(x)} + C;$$

#### Examples

$$1. \int 5 dx = 5x + C; \quad 2. \int x^3 dx = x^4/4 + C; \quad 3. \int 4/x dx = 4 \int x^{-1} dx = 4 \ln x + C;$$

$$4. \int 3e^x dx = 3e^x + C; \quad 5. \int (6x^2 - 4x + 2) dx = 2x^3 - 2x^2 + 2x + C;$$

$$6. \int 4x(x^2 + 1) dx = \int 2(x^2 + 1) 2x dx = \int 2(x^2 + 1) d(x^2 + 1) = (x^2 + 1)^2 + C;$$

$$7. \int 2x(x^2 + 1)^{-1} dx = \int (x^2 + 1)^{-1} 2x dx = \int (x^2 + 1)^{-1} d(x^2 + 1) = \ln(x^2 + 1) + C;$$

$$8. \int e^{4x+3} dx = 1/4 \int e^{4x+3} 4 dx = 1/4 \int e^{4x+3} d(4x+3) = 1/4 e^{4x+3} + C.$$

#### Exercises

$$1. \int (2x^3 - 3x^2 + 4x + 3) dx. \quad 2. \int (4x^4 - 6x^2 + 2x + 7) dx. \quad 3. \int (4x + 1/x) dx. \quad 4. \int \sqrt{x} dx.$$

$$5. \int \frac{4+x}{x} dx. \quad 6. \int \frac{2x-3}{x^4} dx. \quad 7. \int \frac{1}{x^{1/2}} dx. \quad 8. \int \frac{dx}{4x^4}. \quad 9. \int (3x^2 - 2/x) dx.$$

$$10. \int (x+3)^{10} dx. \quad 11. \int (x+3)^{-1} dx. \quad 12. \int 6(6x+3)^2 dx. \quad 13. \int 2(2x-3)^{-2} dx. \quad 14. \int (x+3)^2 dx.$$

$$15. \int 2x(x^2+3)^2 dx. \quad 16. \int x(x^2+3)^2 dx. \quad 17. \int 3x^2(x^3+3)^2 dx. \quad 18. \int x^2(x^3+3)^2 dx.$$

$$19. \int e^{2x} dx. \quad 20. \int 2e^x(2e^x+1) dx. \quad 21. \int e^x(3e^x+1) dx. \quad 22. \int 3e^x(2e^x+1) dx.$$

$$23. \int \frac{1}{5x+4} dx. \quad 24. \int \frac{x}{3x^2+1} dx. \quad 25. \int \frac{x}{1+x^2} dx. \quad 26. \int \frac{3}{2-x} dx.$$

Find  $f(x)$  if

27.  $f'(x) = 2x - 3$ , and  $f(0) = 5$ . 28.  $f'(x) = 6x^2 - 4x$ , and  $f(0) = 3,000$ .

29.  $f'(x) = 20x^{1/2}$ , and  $f(1) = 40$ . 30.  $f'(x) = 2x^{-2} + 3x^{-1}$ , and  $f(1) = 0$ .

31. Marginal cost function is given by  $C'(x) = 3x^2 - 24x + 53$ , and the fixed cost at 0 output are \$ 30,000. What is the cost for manufacturing of 4,000 items?

32. The weekly marginal revenue from the sale of  $x$  pairs of shoes is given by

$$R'(x) = 40 - 0.02x + 200/x + 1.$$

Besides  $R(0) = 0$ . Find the revenue function. Find the revenue from the sale of 1,000 shoes.

33. The weekly marginal cost of producing  $x$  pairs of shoes is  $C'(x) = 12 + 500/(x + 1)$ , and the weekly fixed costs are \$ 2,000. Find the cost function. What is the average cost per one pair of shoes if 1,000 pair of shoes are produced?

## Definite Integral

Suppose  $\int f(x)dx = F(x)$ . Then the definite integral is defined as  $\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$ .

### Examples

1.  $\int_1^3 5 dx = 5x \Big|_1^3 = 15 - 5 = 10$ ; 2.  $\int_1^2 x^3 dx = x^4/4 \Big|_1^2 = 16/4 - 1/4 = 4 - 0.25 = 3.75$ ;

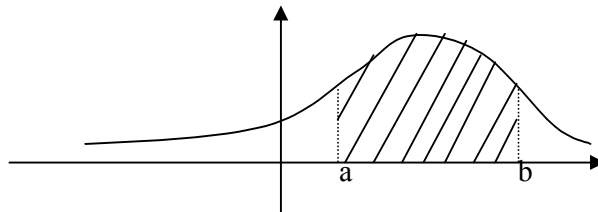
3.  $\int_1^3 4/x dx = 4 \ln x \Big|_1^3 = 4 \ln e^3 - 4 \ln e^1 = 12 - 4 = 8$ ; 4.  $\int_0^2 3 e^x dx = 3 e^x \Big|_0^2 = 3e^2 - 3e^0 = 3e^2 - 3$ .

### Exercises

1.  $\int_1^2 (6x^2 - 4x + 2) dx$ . 2.  $\int_0^1 4x(x^2 + 1) dx$ . 3.  $\int_{-1}^2 2x(x^2 + 1)^{-1} dx$ . 4.  $\int_0^1 e^{4x+3} dx$ .

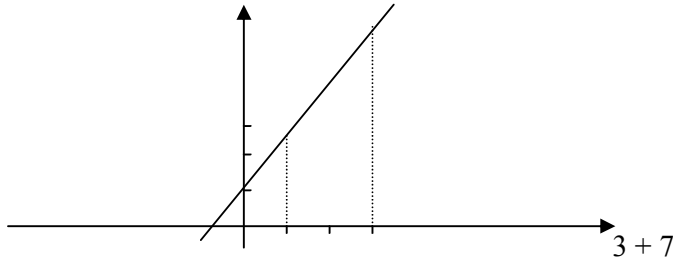
### Finding area

Area under  $y = f(x)$  from  $x = a$  to  $x = b$  is  $S = \int_a^b f(x) dx$ .



### Examples

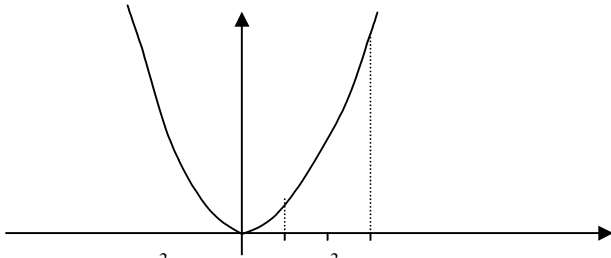
1. Find the area under  $y = 2x + 1$  for  $1 \leq x \leq 3$ .



**Solution 1.**  $f(1) = 3$ ,  $f(3) = 7$ ,  $h = 3 - 1 = 2$ , then  $S = \frac{3+7}{2} \cdot 2 = 10$ .

**Solution 2.**  $S = \int_1^3 (2x + 1) dx = (x^2 + x) \Big|_1^3 = 3^2 + 3 - 1^2 - 1 = 10$ .

2. Find the area under  $y = x^2$  for  $1 \leq x \leq 3$ .



**Solution.**  $S = \int_1^3 x^2 dx = x^3 / 3 \Big|_1^3 = 27 / 3 - 1 / 3 = 26 / 3$ .

### Exercise

1. Find the area under  $y = x^3$  for  $1 \leq x \leq 3$ .

2. Find the area under  $y = 0.5x^2 + 2$  for  $1 \leq x \leq 3$ .