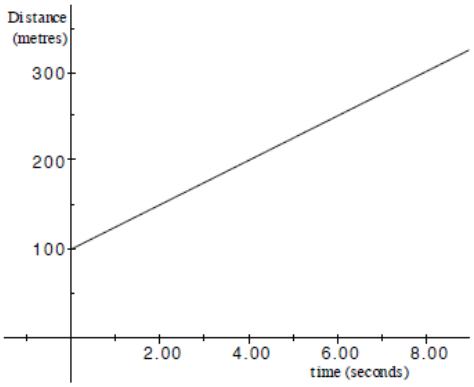

Exercise 1.1

How far is the motorist in Figure 1 away from home at time $t = 0$ and at time $t = 6$?

Exercise 1.2

How far does the motorist travel in the first two seconds (ie from time $t = 0$ to time $t = 2$)? How far does the motorist travel in the two second interval from time $t = 3$ to $t = 5$? How far do you think the motorist would travel in any two second interval of time?

Exercise 1.3

How far does the motorist travel in the two seconds from time $t = 60$ to time $t = 62$?

How far does the motorist travel in the two second interval from time $t = 62$ to $t = 64$?

Exercise 2.1 (You will find this exercise easier to do if you use graph paper.)

Draw a careful graph of the function $f(x) = x^2$. Draw the tangents at the points $x = 1$, $x = 0$ and $x = -0.5$. Find the slopes of these lines by picking two points on them and using the formula

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}.$$

These slopes are the (approximate) values of $f'(1)$, $f'(0)$ and $f'(-0.5)$ respectively.

Exercise 2.2

Repeat Exercise 2.1 with the function $f(x) = x^3$.

Exercises 3.1

Differentiate the following functions:

a. $f(x) = x^4$ **b.** $y = x^{-7}$ **c.** $f(u) = u^{2.3}$

d. $f(t) = t^{-\frac{1}{3}}$ **e.** $f(t) = t^{\frac{22}{7}}$ **f.** $g(z) = z^{-\frac{3}{2}}$

g. $y = t^{-3.8}$ **h.** $z = x^{\frac{3}{7}}$

Exercise 3.2

Express the following as powers and then differentiate:

a. $\frac{1}{x^2}$ b. $t\sqrt{t}$ c. $\sqrt[3]{x}$

d. $\frac{1}{x^2\sqrt{x}}$ e. $\frac{1}{x\sqrt[4]{x}}$ f. $\frac{s^3\sqrt{s}}{\sqrt[3]{s}}$

g. $\frac{1}{u^3}$ h. $\frac{t}{t^2\sqrt{t}}$ i. $x^{\frac{1}{2}}\frac{\sqrt{x}}{x}$

Exercise 3.3

Find the equation of the line tangent to the graph of $y = \sqrt[3]{x}$ when $x = 8$.

Exercise 3.4

Differentiate the following functions:

a. $f(x) = 5x^2 - 2\sqrt{x}$ b. $y = 2x^{-7} + \frac{3}{x^2}$ c. $f(t) = 2.5t^{2.3} + \frac{t}{\sqrt{t}}$

d. $h(z) = z^{-\frac{1}{3}} + 5z$ e. $f(u) = u^{\frac{5}{3}} - 3u^{-7}$ f. $g(z) = 8z^{-2} - \frac{5}{z}$

g. $y = 5t^{-8} + \frac{t}{\sqrt{t}}$ h. $z = 4x^{\frac{1}{7}} + 2x^{-\frac{1}{2}}$

Exercise 3.5

Use the product rule to differentiate the functions below:

a. $f(x) = (4x^3 + 2)(1 - 3x)$

b. $g(x) = (x^2 + x + 2)(x^2 + 1)$

c. $h(x) = (3x^3 - 2x^2 + 8x - 5)(x^2 - 2x + 4)$

d. $f(s) = (1 - \frac{1}{2}s^2)(3s + 5)$

e. $g(t) = (\sqrt{t} + \frac{1}{t})(2t - 1)$

f. $h(y) = (2 - \sqrt{y} + y^2)(1 - 3y^2)$

Exercise 3.6

If $r = (t + \frac{1}{t})(t^2 - 2t + 1)$, find the rate of change of r with respect to t when $t = 2$.

Exercise 3.7

Find the slope of the tangent to the curve $y = (x^2 - 2x + 1)(3x^3 - 5x^2 + 2)$ at $x = 2$.

Exercise 3.8

Use the Quotient Rule to find derivatives for the following functions:

a. $f(x) = \dfrac{x-1}{x+1}$ b. $g(x) = \dfrac{2x+3}{3x-2}$

c. $h(x) = \dfrac{x^2+2}{x^2+5}$ d. $f(t) = \dfrac{2t}{1+2t^2}$

e. $f(s) = \dfrac{1+\sqrt{s}}{1-\sqrt{s}}$ f. $h(x) = \dfrac{x^2-1}{x^3+4}$

g. $f(u) = \dfrac{u^3+u-4}{3u^4+5}$ h. $g(t) = \dfrac{t(t+6)}{t^2+3t+1}$

Exercise 3.9

Differentiate the following functions using the composite function rule.

a. $(2x+3)^2$ **b.** $(x^2+2x+1)^{12}$ **c.** $(3-x)^{21}$

d. $(x^3-1)^5$ **e.** $f(t) = \sqrt{t^2-5t+7}$ **f.** $g(z) = \frac{1}{\sqrt{2-z^4}}$

g. $y = (t^3 - \sqrt{t})^{-3.8}$ **h.** $z = (x + \frac{1}{x})^{\frac{3}{7}}$

Exercise 3.10

Differentiate the functions below. You will need to use both the composite function rule and the product or quotient rule.

a. $(x+2)(x+3)^2$ **b.** $(2x-1)^2(x+3)^3$ **c.** $x\sqrt{(1-x)}$

d. $x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}$ **e.** $\dfrac{x}{\sqrt{1-x^2}}$

Exercise 3.11

Differentiate the following functions.

a. $f(x) = \ln(2x^3)$ **b.** $f(x) = e^{x^7}$ **c.** $f(x) = \ln(11x^7)$

d. $f(x) = e^{x^2+x^3}$ **e.** $f(x) = \log_e(7x^{-2})$ **f.** $f(x) = e^{-x}$

g. $f(x) = \ln(e^x + x^3)$ **h.** $f(x) = \ln(e^x x^3)$ **i.** $f(x) = \ln\left(\dfrac{x^2+1}{x^3-x}\right)$

Exercise 3.12

Differentiate the following:

a. $\cos 3x$ **b.** $\sin(4x+5)$ **c.** $\sin^3 x$ **d.** $\sin x \cos x$ **e.** $x^2 \sin x$

f. $\cos(x^2+1)$ **g.** $\dfrac{\sin x}{x}$ **h.** $\sin\dfrac{1}{x}$ **i.** $\tan(\sqrt{x})$ **j.** $\dfrac{1}{x}\sin\dfrac{1}{x}$

Exercise 4.1 Find the maximum and the minimum of the function $f(x) = x^4 - 2x^2$ for $-1 \leq x \leq 2$

Exercise 4.2 Maximise the function $g(t) = te^{-t^2}$ for $-2 < t < 2$.

Exercise 4.3 Find the minimum value of $h(u) = 2u^3 + 3u^2 - 12u + 5$ in the region $-3 \leq u \leq 2$.

Exercise 4.4 A farmer wishes to make a rectangular chicken run using an existing wall as one side. He has 16 metres of wire nettting. Find the dimensions of the run which will give the maximum area. What is this area?