Tornike Kadeishvili

1 Sets

"A set is a collection of objects which we call elements"
(Set=Collection?! Tautology).

Examples

1. Class2014={Set of all students in this class}.
2. ISET={Set of all students of ISET}.
3. Set of natural numbers N.
4. Set of whole numbers Z.
5. Set of rational numbers Q.
6. Set of real numbers R.
7. Set of complex numbers C.

1.0.1 Subset

$A \subset X \Leftrightarrow (x \in A \Rightarrow x \in X)$
(there exists no x in A which is not in X).

Examples

1. Class2014$\subset$ ISET.
2. $N \subset Z \subset Q \subset R \subset C$.
3. $\emptyset \subset X$ for any set X.
(there exists no x in $\emptyset$ which is not in X)

1.0.2 Operations on Sets

Union

$$A \bigcup B = \{x,\ x \in A\ or\ x \in B\}.$$

Intersection

$$A \bigcap B = \{x,\ x \in A\ \ and\ \ \in B\}.$$

Subtraction

$$A - B = \{x,\ x \in A\ but\ not\ x \in B\}.$$

Denote by U the Universe (biggest set in consideration)

Complement

$$\sim A = A^c = U - A.$$

1.0.3 Properties of Operations

1. $\emptyset \bigcup A = A$
2. $A \bigcup B = B \bigcup A$
3. $(A \bigcup B) \bigcup C = A \bigcup (B \bigcup C)$
4. $A \bigcup A = A$
5. $A \bigcap U = A$
6. $A \bigcap B = B \bigcap A$
7. $(A \bigcap B) \bigcap C = A \bigcap (B \bigcap C)$
8. $A - B = A \bigcap B^c$
9. $(A^c)^c = A$
10. $A \bigcap A = A$
11. $A \bigcap (B \bigcup C) = (A \bigcap B) \bigcup (A \bigcap C)$
12. $A \bigcup (B \bigcap C) = (A \bigcup B) \bigcap (A \bigcup C)$
13. $A \bigcap A^c = \emptyset$
14. $A \bigcup A^c = U$
15. $(A \bigcup B)^c = A^c \bigcap B^c$
16. $(A \bigcap B)^c = A^c \bigcup B^c$

Exercises

1. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find

$A^c =$
$B^c =$
$A \bigcup B =$
$A \bigcap B =$
$A \bigcup B^c =$
$A^c \bigcap B =$
$(A \bigcup B)^c =$
$A^c \bigcap B^c =$
$(A \bigcap B)^c =$
$A^c \bigcup B^c =$

2. Answer the same question for

$$A = [0, 2], \quad B = [1, 4], \ U = R = (-\infty, \infty).$$

3. Answer the same question for

$$\begin{aligned} &A = \{(x, y) \in R^2, \ 0 \le x \le 2, \ 0 \le y \le 2\}, \\ &B = \{(x, y) \in R^2, \ 1 \le x \le 4, \ 1 \le y \le 4\}, \\ &U = R^2. \end{aligned}$$