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0.1 Mathematical Model of Production

Data: x number of units produced, F the fixed cost, c the production cost of one unit, p the wished selling price of one unit, k the reduction coefficient.

The total cost of production of x units

$$C(x) = c \cdot x + F.$$

The selling price of one unit when x units are produced
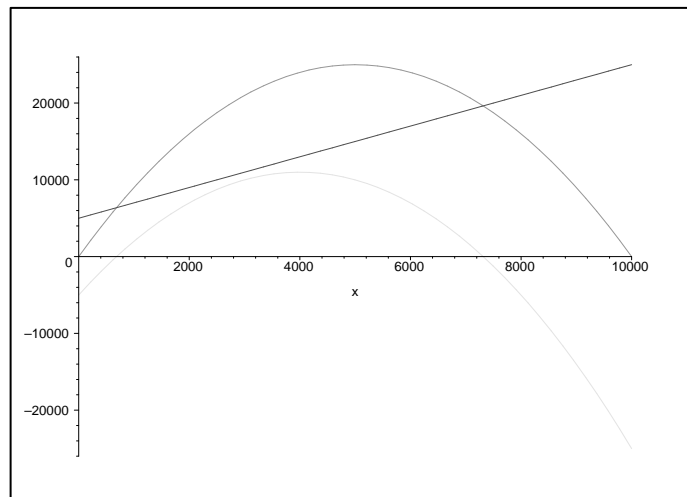
$$p(x) = p - k \cdot x.$$

The total revenue
$$R(x) = x \cdot p(x) = -kx^2 + px.$$

The total profit

$$P(x) = R(x) - C(x) = -kx^2 + px - cx - F = \\ -kx^2 + (p - c)x - F.$$

Both functions $R(x)$ and $P(x)$ are concave quadratic functions, thus they both have maximum.

$> C(x) := 2 * x + 5000; p(x) := 10 - 0.001 * x;$
$> R(x) := x * p(x); P(x) := R(x) - C(x);$
$> plot(R(x), C(x), P(x), x = 0..10000);$

Exercises

1. The total cost of a company per month is given by $C(x) = 2x + 5000$ and the Price - demand function is given by $p(x) = 10 - 0.001x$. Thus the revenue and profit functions are given by

$$R(x) = xp(x) = -0.001x^2 + 10x,$$
$$P(x) = R(x) - C(x) = -0.001x^2 + 8x - 5000.$$

A) How many units should the company manufacture each month to maximize the revenue?

$$R'(x) = -0.002x + 10 = 0, \quad x = 5000$$

B) What is the maximal revenue?

R(5 000)=25 000

C) What is the selling price of one unit when the revenue is maximal?

$p(5000) = 5.$

D) How many units should the company manufacture each month to maximize the profit?

$$P'(x) = -0.002x + 8 = 0, \quad x = 4000.$$

E) What is the maximal profit?

$P(4000) = 11000.$

F) What is the selling price of one unit when the profit is maximal?

$p(4000) = 6.$

G) Find the break-even points.

$$R(x) = C(x), \ P(x) = 0, \ -0.001x^2 + 8x - 5000 = 0, \ x_1 = 683, \ x_2 = 7317.$$

2. The total cost of a company per month is given by $C(x) = 100x + 1000$ and the Price - demand function is given by $p(x) = 300 - 0.1x$.
A) How many TV sets should the company manufacture each month to maximize the revenue?
B) What is the maximal revenue?

C) What is the selling price of one TV set when the revenue is maximal?

D) How many TV sets should the company manufacture each month to maximize the profit?

E) What is the maximal profit?

F) What is the selling price of one TV set when the profit is maximal?

3. Suppose the government decides to introduce additional $ 10 tax for each TV set. Solve the previous problem in this case.