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Boundary and contact problems of elasticity theory for generalized models

Project Summary:

Actuality and novelty of problems. Modern industrial and technological processes apply widely composite materials with complex structure. Therefore the development, investigation and analysis of mathematical models of such materials with appropriate mechanical, thermal, electric and other physical properties has a crucial importance for both fundamental research and practical applications. In particular, invaluable interest for fundamental research represents the investigation of correctness of corresponding mathematical models (namely existence, smoothness, uniqueness and stability of solutions) and development of adequate computing algorithms for a practical implementation of fundamental results.

In the practice of engineering and industrial applications we encounter rather often such composite materials when at different parts of composites are modeled by physical fields of different dimension. As an example we can quote interaction of piezoceramic materials with metallic electrode inclusions which takes into account thermal effects: in the piezoceramic part the physical field is represented by 5-component vector, while in metallic part-by 4-component vector. In such a case we actually have deal with the interaction of physical fields with different dimension in the vicinity of the interfaces. The corresponding mathematical models are described, as a rule, by boundary, boundary-contact and transmission problems for systems of differential equations with partial derivatives. The situation is especially complicated because we have to establish conditions for conjugation of those physical fields with different dimension on the interfaces. Theory of such a general boundary-contact nd boundary-transmission problems is far from being complete.

Purpose of investigation and expected results. Modern engineering constructions represent complex structures assembled of composition materials of different dimension: dimension one (strings, ropes, beams), dimension two (membranes, plates, shells) and dimension three. Therefore a mathematical modeling of such objects require elaboration of adequate mathematical models for each of these elements and their complex investigation. The present project is oriented towards the investigation of such patterns for some modern composite materials and should be therefore minded as the first but crucial step towards modeling of above mentioned complex engineering assemblages. In particular,

a) the project suggests the mathematical modeling of interaction of complex thermopiezoelectric composite materials and the investigation of corresponding boundary and boundary contact problems. Will be established solvability and uniqueness of solutions of static and pseudo-oscillation problems of interaction of thermopiezoelectric composite materials, will be investigated the behavior of solutions in the vicinity of singularity curves, concentration of stresses and their singularity parameters. By numerical experiments for model problems will be revealed dependence of mechanical stresses and the singularity rates of electric fields on the thermo-electro-mechanical characteristics of the material.

b) mathematical modeling of mechanical problems with unilateral constraints for hemitropic materials and a complete mathematical treatment of the corresponding problems will be suggested. Will also be treated contact problems for multi-component elastic hemitropic bodies. Will be obtained

general representation of solutions to problems of static and steady state oscillation of hemitropic elasticity theory. Explicit solutions to problems of static and steady state oscillation for piecewise-homogeneous bodies with spherical interfaces will be written.

c) is planned mathematical treatment of boundary value problems for partial differential equations on hypersurfaces; problems of such type we encounter, for example, in the theory of shallow shells and thin plates, heat conduction by surfaces and many other practical problems.

Areas of possible application. The results of the investigation planned by the project might be applied to the location of stress concentration zones and detection how these zones are influenced by various mechanical, thermal and electric fields in case of static and resonance oscillations. Special importance acquire precise values of stress field singularity parameters in the vicinity of singularities of a material (e.g., in the vicinity of inner cracks), which play a crucial role in prediction of a crack propagation and material failure.

Possible approach of the investigation. Theoretical research of the problems formulated in the project will be implemented with the help of the potential method and the theory of pseudodifferential equations, as well as by means of special and boundary variational inequalities. Due to the complexity of the formulated problems it will be necessary a non-trivial combination of the above mentioned approaches and application of some new combined methods.

For equations on hypersurfaces will be applied Gunter's and Stokes derivatives and their combination, which enable to record important boundary value problems on surfaces in relatively simple form.

To develop rigorous approximation algorithms and obtain appropriate numerical results it is crucial to apply optimal schemes of hybrid boundary-difference and boundary element methods.

Which experience in the treatment of the problem possess the participants of the project. The competence of the participants of the project is justified by their active and fruitful work in the theory of solid mechanics and equations of mathematical physics. They participated and still participate in many international scientific projects. They have tight scientific contacts with leading research centers worldwide and are involved in joint research projects with the colleagues abroad. The participants of the project are rather often invited to international congress, conferences and symposia as plenary or sectional speakers (cf. their CVs).

The topic of the project represents the continuation and enhancement of the topics treated in the papers published by the participants. Their knowledge and experience a reliable indicator that all problems listed in the project will be implemented properly.