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EFFECT OF PRANDTL NUMBERS ON THE TRANSITIONS OF DIVERGING AND CONVERGING HEAT-CONDUCTING FLOWS IN AN ANNULUS

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Dedicated to the memory of Academician Vakhtang Kokilashvili

Abstract. In this paper, the effect of Prandtl numbers on the transition of a heat-conducting flow subjected to the action of a radial fluid through the horizontal permeable cylinder walls is investigated. The cylinders are heated up to different temperatures and driven by a constant azimuthal pressure gradient.

Introduction

The stability of a heat-conducting flow between porous cylinders heated up to different temperatures and driven by a constant azimuthal pressure gradient can be affected by various parameters, including the Prandtl number (Pr). As is known, this number is dimensionless and represents the ratio of kinematic viscosity to thermal conductivity. When the Prandtl number is low, it means that thermal diffusivity dominates over the kinematic viscosity. In such cases, thermal effects are more significant than the viscous effects. Conversely, when the number Pr is high, kinematic viscosity dominates over the thermal diffusivity and the flow is more subject to the viscous effects. In this case, the heat transfer may not be as efficient and the stability behavior could differ from that observed for low Pr.

The influence of number Pr on the stability of a heat-conducting flow between concentric cylinders was investigated by various authors (see e.g., [1–3, 5] and references therein). For instance, in [2], the effect of Pr on the stability of Couette–Taylor flow by rotating cylinders is investigated in the presence of gravity. Also, the influence of Pr neglecting the effect of gravity and taking into account radial temperature gradient and constant azimuthal pressure gradient on the stability of flow between horizontal cylinders is studied in [3]. In [1], it has been found that the flow for a low Pr under axisymmetric disturbance is more stable as compared with the flow under non-axisymmetric disturbance. When the gap between the cylinders is relatively small, the flow under non-axisymmetric disturbance is most stable for the fluids having high Pr. The instability mechanisms of the mixed convective flow and its dependence on the Prandtl number are studied in [5]. In these papers, the investigation of stability of the main flow is carried out by the linear approximations. Based on the nonlinear theory of hydrodynamic stability, in [12], for Pr = 0.71 (for air and gases), the bifurcations of Dean's flow between horizontal porous cylinders heated up to different temperatures, with a radial flow, and driven by a constant azimuthal pressure gradient, were studied.

In the present paper, we are interested in how a high Prandtl number, for instance, Pr = 7 (for liquid) affects the transition to possible complex modes in the main flow, which appearance precedes the development of high instability.

1. Formulation of the Problem

The heat-conducting flow between horizontal porous concentric cylinders of radii R_1, R_2, and temperature T_1, T_2 of the inner and outer cylinders, respectively, is maintained by a constant azimuthal pressure gradient and in the presence of a diverging or converging flow through the horizontal cylinder walls and is described by the Navier–Stokes system and a continuity equation in the cylindrical coordinates r, θ, z with a velocity vector $v'(v'_r, v'_\theta, v'_z)$. Under the above assumptions, there exists

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the following exact solution of the Navier–Stokes equations for the velocity V_0, temperature T_0, and pressure Π_0 (see [12]):

$$\begin{gathered}
V_0 = \{u_0(r), v_0(r), 0\}, \quad T_0 = c_1 + c_2 r^{\varkappa P_r}, \\
u_0(r) = \frac{R_1 U_0}{r}, \quad v_0(r) = \begin{cases} \dfrac{K}{\varkappa}\Big(a r^{\varkappa+1} + \dfrac{b}{r} - r\Big), & \varkappa \neq -2, \\ \dfrac{K}{2}\Big(\dfrac{a_1 \ln r + b_1}{r}\Big), & \varkappa = -2, \end{cases} \\
\frac{\partial \Pi_0}{\partial r} = \frac{\rho(u_0^2 + v_0^2)}{r},
\end{gathered} \tag{1.1}$$

where

$$\begin{gathered}
K = \frac{1}{2\rho\nu}\Big(\frac{\partial \Pi_0}{\partial \theta}\Big)_0 = \text{const}, \quad a = \frac{R^2 - 1}{(R^{\varkappa+2} - 1)R_1^{\varkappa}}, \quad a_1 = \frac{R_1^2(R^2 - 1)}{\ln R}, \\
b = \frac{R_2^2(R^{\varkappa} - 1)}{R^{\varkappa+2} - 1}, \quad b_1 = -\frac{R_1^2 \ln R_2 - R_2^2 \ln R_1}{\ln R}, \\
c_1 = \frac{T_1 R^{\mathrm{Pr}\,\varkappa} - T_2}{R^{\varkappa\,\mathrm{Pr}} - 1}, \quad c_2 = \frac{T_2 - T_1}{R_1^{\varkappa\,\mathrm{Pr}}(R^{\varkappa\,\mathrm{Pr}} - 1)}, \quad R = \frac{R_2}{R_1},
\end{gathered}$$

$\varkappa = \frac{U_0 R_1}{\nu}$ is the radial Reynolds number, U_0 is the radial velocity of the flow through the wall of the inner cylinder, $\mathrm{Pr} = \frac{\nu}{\chi}$ is the Prandtl number, ν, χ, β are, respectively, the coefficients of kinematic viscosity, thermal diffusion and thermal expansion, ρ is a density of the fluid, the radial flow is inward for $\varkappa < 0$ (converging flow), outward for $\varkappa > 0$ (diverging flow), and so the following boundary conditions:

$$\begin{gathered}
v'_r\big|_{r=R_1} = U_0, \quad v'_\theta = 0, \quad v'_z = 0, \quad T' = T_1 \ \ (r = R_1), \\
v'_r\big|_{r=R_2} = \frac{U_0}{r}, \quad v'_\theta = 0, \quad v'_z = 0, \quad T' = T_2 \ \ (r = R_2)
\end{gathered} \tag{1.2}$$

are satisfied.

The flow (1.1)–(1.2) with the velocity vector V_0, temperature T_0 and pressure Π_0 will be called the main stationary flow. In the sequel, it will always be assumed that the velocity, temperature and pressure components are periodic with z and θ, with the periods $2\pi/\alpha$ and $2\pi/m$, respectively, $\alpha \geq 0$, $m = 0, 1$.

Let the perturbed state be taken as

$$v' = V_0 + V(v_r, v_\theta, v_z), \quad T' = T_0 + T, \quad \Pi' = \Pi_0 + \Pi. \tag{1.3}$$

We introduce the dimensionless variables for time, length, velocity, temperature and pressure and denote them by S, R_2, SR_2, $T_2 - T_1$, $\nu\rho S$, respectively, where the rotation shear S is denoted by $\frac{V_m}{d}$, V_m is an average velocity in the azimuthal direction, $d = R_2 - R_1$. Under these assumptions, we obtain the following nonlinear system of perturbation equations (see [12]):

$$\frac{\partial v}{\partial t} + Nv - \frac{1}{\mathrm{Re}} Mv + \frac{1}{\mathrm{Re}} \nabla_1 \Pi = -\mathcal{L}(v, v), \quad (\nabla_1, rv) = 0, \quad v\big|_{r=1/R,1} = 0, \tag{1.4}$$

where

$$\begin{gathered}
Mv = \Big\{\Delta_1 v_r - \frac{1-\varkappa}{r^2} v_r - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}, \Delta_1 v_\theta - \frac{1+\varkappa}{r^2} v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}, \Delta_1 v_z, \frac{1}{\mathrm{Pr}} \Delta_1 T\Big\}, \\
Nv = \omega_1 \frac{\partial v}{\partial \theta} + \Big\{\mathrm{Ra}\,\omega_2 - 2\omega_1 v_\theta, -g_1 v_r, 0, \frac{g_2}{\mathrm{Pr}} v_r,\Big\}, \\
\mathcal{L}(v, v) = \Big\{(v, \nabla_1) v_r - \frac{v_\theta v_\theta}{r}, \ (v, \nabla_1) v_\theta + \frac{v_r v_\theta}{r}, (v, \nabla_1) u_z, (v, \nabla_1) T_1\Big\}, \\
\Delta_1 = \frac{\partial^2}{\partial r^2} + \frac{1-\varkappa}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}, \quad \nabla_1 = \Big\{\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}, \frac{\partial}{\partial z}, 0\Big\},
\end{gathered}$$

$$\mathrm{Re} = \frac{V_m d}{\nu} \text{ is the azimuthal Reynolds number },$$

$$\mathrm{Ra} = \frac{\beta(T_2 - T_1)}{2} \text{ is the Rayleigh number},$$

$$V_m = K\,\frac{R_1 R^2}{R-1}\,D(R), \quad D(R) = \frac{R^{\varkappa}-1}{R^{\varkappa+2}-1}\,\ln R - \frac{\varkappa(R^2-1)}{2R^2(\varkappa+2)}\,,$$

$$\omega_1 = \frac{v_0(r)}{r} = \lambda g(r) + g_0(r), \quad \omega_2 = \omega_1^2 r,$$

$$g(r) = \frac{d}{R_2}\,\frac{D1(R)r^{\varkappa+2} + D2(R) - r^2}{rD(R)}\,, \quad g_0(r) = D3(R)r^{\varkappa+1} + \frac{D4(R)}{r}\,,$$

$$g_1(r) = -\Big(\frac{dv_0}{dr} + \frac{v_0}{r}\Big) = -\Big(\frac{d}{R_2}\,\frac{D1(R)\,(\varkappa+2)r^{\varkappa} - 2}{D(R)}\Big), \quad g_2(r) = \frac{\varkappa\,\mathrm{Pr}^2\,R^{\varkappa\,\mathrm{Pr}}}{R^{\varkappa\,\mathrm{Pr}} - 1}\,r^{\varkappa\,\mathrm{Pr}-1},$$

$$D1(R) = \frac{(R^2-1)R^{\varkappa}}{R^{\varkappa+2}-1}\,, \quad D2(R) = 1 - D1(R), \quad D3(R) = \frac{R^{\varkappa}-1}{R^{\varkappa+2}-1}\,.$$

Problem (1.4) is written in terms of the Boussinesq approximation [4], which is based on the assumption that the thermal expansion coefficient is small. The flow with the velocity vector V_0, temperature T_0 and pressure Π_0 will be called the main stationary flow and defined by the parameters Re, R, Ra, Pr, $\varkappa$, α, m.

Our aim is to investigate oscillatory regimes arising in a small neighborhood of the point of intersection of neutral curves corresponding to the rotationally symmetric and oscillatory instability of the main stationary flow (1.1)–(1.2).

2. Transitions to Complex Regimes

In studying the transition of flow (1.1)–(1.2) to complex regimes, particular attention was given to identifying the points of intersection of neutral curves. At these points, a strong interaction between the vortices and azimuthal flows and also the emergence of rather complex regimes of a fluid motion are expected.

To construct neutral curves, we assume that the perturbations V, temperature T and pressure Π are infinitely small. The neutral curves which correspond to the bifurcation of vortices and azimuthal waves are found by solving the following spectral problems:

$$(M - \mathrm{Re}\,N)\Phi_0 = \nabla_1 p_0, \quad (\nabla_1, r\Phi_0) = 0, \quad \Phi_0\big|_{r=1,R} = 0, \tag{2.1}$$

and

$$(M - \mathrm{Re}\,N - ic\,\mathrm{Re})\Phi_1 = \nabla_1 p_1, \quad (\nabla_1, r\Phi_1) = 0, \quad \Phi_1\big|_{r=1,R} = 0, \tag{2.2}$$

where

$$\Phi_0 = \big\{u_0(r), v_0(r), iw_0(r), \tau_0(r)\big\}e^{i\alpha z}, \quad p_0 = q_0(r)e^{i\alpha z}, \tag{2.3}$$

$$\Phi_1 = \big\{u_1(r), v_1(r), w_1(r), \tau_1(r)\big\}e^{-i(m\theta+\alpha z)}, \quad p_1 = q_1(r)e^{-i(m\theta+\alpha z)}, \tag{2.4}$$

c is an unknown frequency of neutral azimuthal waves.

The eigenvalues problems (2.1)–(2.4) have been solved by the shooting method for fixed $\varkappa$, α, R, m, Pr. Thus for the fixed values of these parameters, we have investigated the dependence of a critical value of numbers Re, Ra and the neutral mode of frequency c corresponding to the emergence of vortices and azimuthal waves. Further, using the Newton method, one can calculate with sufficient exactness the values Re_0, Ra_0 and c_0 corresponding to the point of intersection of neutral curves.

Let $(\mathrm{Ra}_0, \mathrm{Re}_0)$ be the points lying on the plane of parameters $(\mathrm{Ra}, \mathrm{Re})$ and corresponding to the intersection of neutral curves corresponding to the monotone ($m = 0$) axisymmetric and oscillatory non-axisymmetric three-dimensional loss of stability of the main stationary flow (1.1)–(1.2).

The flow regimes appearing in a small neighborhood of the point of intersection of neutral curves of axisymmetric and oscillatory instabilities are investigated by the analysis of dynamical system of amplitude equations which are used for a wide class of problems with a cylinder symmetry (see, e.g., [6–13] and references therein). The system describes the nonlinear interaction of axisymmetric and oscillatory three-dimensional flow regimes and is a system of three complex differential equations of the first order. The $G = SO(2) * O(2)$ symmetry enables us to reduce the six-dimensional amplitude system to the four-dimensional system with free parameters σ, μ (the damping decrements of the monotone and oscillatory perturbations, respectively). The system is called a motor subsystem. As it was shown in [6], the motor subsystem has equilibria lying on the invariant subspaces and also equilibria of general state. Following this monograph, the motor subsystem has the following type of equilibria:

(i) Equilibria lying on the invariant planes:
  (a) The main flow;
  (b) Vortices;
  (c) Purely azimuthal waves;
  (d) Spiral Waves;
  (e) A pair of mixed azimuthal waves;
(ii) General equilibria not lying on the invariant planes: two-frequency quasiperiodic modes;
(iii) Limit cycles: three-frequency quasiperiodic modes.

We present here the scheme of equilibria bifurcations of a motor subsystem for the main stationary flow (1.1)–(1.2). This scheme allows us to show the influence of the magnitude for $\mathrm{Pr} = 7$ (liquid) as comparison with the transition to complex flows after the loss of stability of the main stationary flow in case $\mathrm{Pr} = 0.71$ (air and gases).

In Figures 1–3, we present the scheme of equilibria bifurcations of the motor subsystem, which we consider the most interesting and allowing us to judge about the transition character of the system under consideration.

The single lines show symmetric equilibria, the double lines indicate a connected pair of equilibria. Stable equilibria are drawn by solid lines and unstable equilibria by dotted lines. The circles are the points at which the motor subsystem limit cycles bifurcate.

We present here several of our results obtained for $R = 2$ (the radius of the outer cylinder is two times greater than that of the inner one), $Pr = 7$, $m = 0, 1$, $\alpha \in [4, 8]$ (short-wave axially directed perturbations) and for small absolute values of the radial Reynolds number.

In Figure 1, we consider the scheme of transitions when the main flow is directed from the outer cylinder to the inner cylinder ($\varkappa = -0.5$), the Rayleigh number $\mathrm{Ra}_0 = 3.8654$ (temperature of the outer cylinders is higher than that of the inner cylinder), $\alpha = 4$ (perturbation is $\pi/2$ periodical in the axial direction), $\mathrm{Re}_0 = 7.102$ (the azimuthal Reynolds number), $c_0 = 3.5788$ (unknown frequency of neutral azimuthal waves).

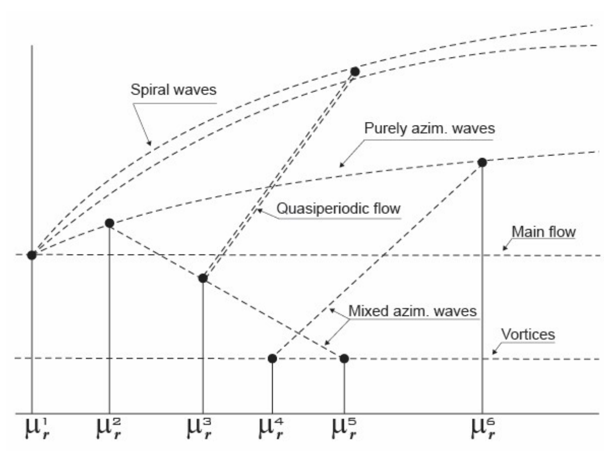

FIGURE 1. $\sigma < 0$, $\alpha = 4$, $\varkappa = -0.5$, $\mathrm{Ra}_0 = 3.8654$, $\mathrm{Re}_0 = 7.1022$, $c_0 = 3.5788$. Bifurcation values: $\mu_r^1 = 0$, $\mu_r^2 = 0.1232$, $\mu_r^3 = 1.65$, $\mu_r^4 = 1.87$, $\mu_r^5 = 3.213$, $\mu_r^6 = 3.722$.

The main flow exists for $\sigma < 0$ and for any value of a free parameter μ_r (μ_r is the real part of a damping decrement of oscillatory perturbation μ). This flow is unstable. For $\mu_r = \mu_r^1 = 0$, from the main flow bifurcate simultaneously unstable spiral waves, pure azimuthal waves and vortices. For $\mu_r = \mu_r^2$, from purely azimuthal waves branch off mixed azimuthal waves, these waves exist in the range $\mu_r^2 < \mu_r < \mu_r^5$ and disappear for $\mu_r = \mu_r^5$ merging with unstable vortices. For $\mu_r = \mu_r^3$, from mixed azimuthal waves bifurcates a quasi-periodic flow, which merges with the spiral flow. From the unstable vortices $\mu_r = \mu_r^4$ bifurcate mixed azimuthal waves merging with unstable pure azimuthal waves.

In contrast to the case for Pr = 0.71 (see [12], Figure 3), when after the loss of stability in the main flow we have both the bifurcation cycles branching from the equilibria and also several stable equilibria, which can be observed in the experiments as hysteresis states, whereas in the case for Pr = 7, after the loss of stability in the main flow the bifurcation of cycles does not take place; there are only unstable equilibria. Consequently, in this case, with the corresponding values of the problem parameters, after the loss of stability of the main flow, there immediately arise quite complex regimes (see [13]).

The following transition diagram (Figure 2) describes the case in which $\varkappa = -3$ (converging flow), Rayleigh number $\mathrm{Ra}_0 = 24.2054$ (temperature of the outer cylinders is higher than that of the inner cylinder), $\alpha = 8$ (perturbation is $\pi/4$ periodical in the axial direction), $\mathrm{Re}_0 = 17.985$ (the azimuthal Reynolds number), $c_0 = 3.79358$ (an unknown frequency of neutral azimuthal waves).

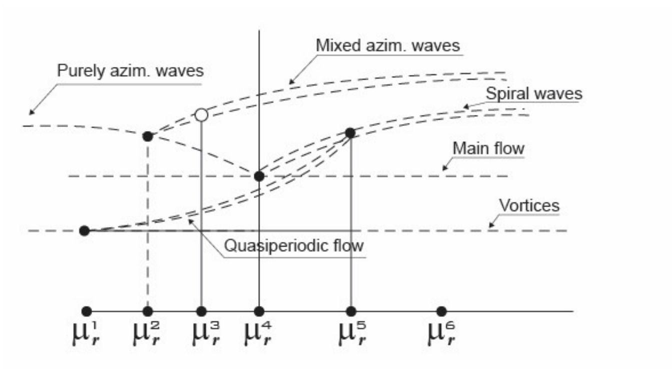

FIGURE 2. $\sigma > 0$, $\alpha = 8$, $\varkappa = -3$, $\mathrm{Ra}_0 = 24.20541$, $\mathrm{Re}_0 = 17.985$, $c_0 = 3.793$. Bifurcation values: $\mu_r^1 = -0.4038$, $\mu_r^2 = -0.086$, $\mu_r^3 = -0.0666$, $\mu_r^4 = 0$, $\mu_r^5 = 0.14588$.

The main stationary flow exist for any value of the parameter μ. For $\sigma > 0$, it is unstable. For $\mu_r = \mu_r^4 = 0$, from this flow, in a subcritical space, bifurcate unstable pure azimuthal waves and spiral waves, and for $\sigma = 0$, bifurcate stable vortices. For $\mu_r = \mu_r^2$, from purely azimuthal waves bifurcate unstable mixed azimuthal waves and for $\mu_r = \mu_r^1$, from vortices bifurcates an unstable guasiperiodic flow, which exists in the range $\mu_r^1 < \mu_r < \mu_r^5$ and disappears for $\mu_r = \mu_r^5$ merging with an unstable spiral flow. For $\mu_r = \mu_r^2$, from mixed azimuthal waves there also bifurcate unstable cycles.

In this case, as compared with the case for $P_r = 0.71$ (see [12], Table 1), for sufficiently large values of $\alpha = 8$, i.e., for the $\pi/4$ periodical axially directed perturbations, no crossing points of neutral curves are found, whereas for Pr = 7, we observe the crossing of neutral curves; this indicates that $\pi/4$ is an axially directed perturbation generating vortices and azimuthal waves that are interacting and with a high probability, there arise various regimes including likewise the complex ones [6].

The scheme in Figure 3 shows the transitions when the flow is directed from the inner cylinder to the outer cylinder ($\varkappa = 3$), Rayleigh number $\mathrm{Ra}_0 = 17.616$ (temperature of the outer cylinders is higher than that of the inner cylinder), $\alpha = 4$ (perturbations are $\pi/2$ periodical in the axial direction), $\mathrm{Re}_0 = 16.5196$ (the azimuthal Reynolds number), $c_0 = 2.68958$ (an unknown frequency of neutral azimuthal waves) In this case, we have the scheme of transition for a diverging flow. From the main flow bifurcate unstable pure azimuthal waves and spiral waves. From the spiral waves, for $\nu_r = \nu_r^2$, bifurcates an unstable quasiperiodic flow. There are no other bifurcations.

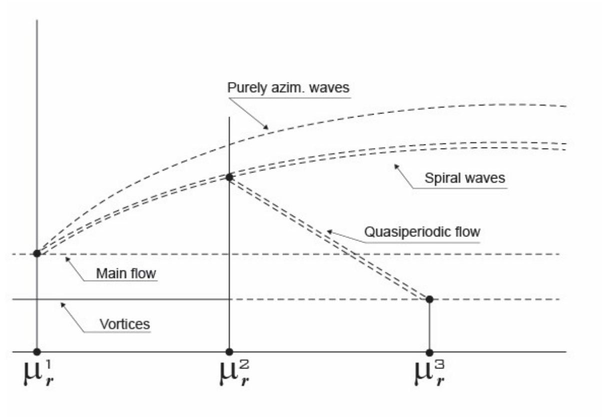

FIGURE 3. $\sigma > 0$, $\alpha = 4$, $\varkappa = 3$, $\mathrm{Ra}_0 = 17.616$, $\mathrm{Re}_0 = 16.5196$, $c_0 = 2.68995$. Bifurcation values: $\mu_r^1 = 0$, $\mu_r^2 = 2.39146$, $\mu_r^3 = 4.12$.

As compared with the case $\mathrm{Pr} = 0.71$ (see [12], Figure 2), when there exist a stable vortex, several unstable bifurcations and also the bifurcation of cycles branching from equilibria, in case $\mathrm{Pr} = 7$, we have a simple scheme of transition of the main flow to complex modes.

3. Conclusion

The paper studies the influence of the magnitude of the Prandtl number $\mathrm{Pr} = 7$ on the transition of the heat-conducting flow between the porous horizontal cylinders as a result of the loss of stability of the main stationary flow. The cylinders are heated up to different temperatures and driven by a constant azimuthal pressure gradient. By the results of numerical analysis, the presented schemes allow us to see the influence of a magnitude of the Prandtl number, or more specifically, of the numbers $\mathrm{Pr} = 0.71$ (for air and gases) and $\mathrm{Pr} = 7$ (for liquid), on the transitions of the main flow to the complex regimes that differ from each other and of importance in experiments.

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