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MULTI-STEP DIFFERENTIAL TRANSFORM METHOD FOR SOLVING THE EPIDEMIC MODEL WITH PINE WILT DISEASE

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Abstract. To solve the system of differential equations which represents the present pine wilt disease epidemic model, we use the computational algorithms like the differential transform method, and it is a multi-step approach.

Description of proposed transformations is constructed at first. Then ODEs of the proposed Epidemic Model are utilized to build the mathematical model.

Obtaining the reliable approximate solutions in the larger domain had been succeeded in the multi-step differential transform method than in the differential transform method, since local convergence of the Taylor series assures that an initial condition is updated in each subdomain, so the error is minimized in a larger domain.

The efficiency of this method is discussed and a reasonable solution for differential transform method, as well as for multi-step differential transform method is obtained. Further, the obtained solutions are compared with the RK4 method, they numerically coincide.

1. Introduction

The terminology "mathematical modelling" is utilised when describing a system by using mathematical notions and language and creating an hypothetical mathematic representation. Mathematical modelling can be seen in numerous fields to convert real world scenarios into just numbers and mathematical data such as Mona Lisa painting in real world is a collection of golden ratio rectangles embraced by Leonardo da Vinci, or the song detecting service Shazam acquired by Apple inc. was developed to creat a mathematical model of three-dimensional song spectrogram in a. 2d star map and applying hash-tables in mathematical world. The way we achieve this is the use of numerous variables to represent internal states, inputs, and outputs, and collection of equations and inequalities to express their interaction. Mathematical modelling in epidemiology escalates our understanding of the system intensifying the spread of pandemic diseases such as Peach yellow which broke out in Philadelphia, United States, in the final decade of the 18th century and spread dangerously over the north-eastern states, eventually arriving Michigan shadowing bankruptcy among the farms and villages in its rouse. Application of mathematical modelling knowledge in understanding plant disease trends, develop as well as assay strategies to combat it can help us avoid global food outage and fulfil the one of the humans' basic need. The numerous diseases which infect vascular system of plants is regarded as wilt diseases. Fungi, bacteria, and nematodes attack on plants can lead to their instant elimination. Plants do possess viruses. In woody plants wilt diseases can be categorized into two parts: those that begin at the branches and those with the roots. The ones which kick start at the branches are likely to begin with the pathogens that feed on leaves or bark and the others initiate infecting by lesioning, or pathogens paving their way directly into the roots, few roll out to other plants through root scion. In 1997, K. Fukuda [5] gave a speculation on the physiological mechanism of the symptom progress and the opposition mechanism in the pine wilt disease after assessing few pathological experiments. Awan et al. [4] presented a mathematical formulation of pine wilt disorder by accounting direct and indirect transmission. Shi and Song [24] drew up and scrutinised a pine wilt disease model regarding invariance of non-negativity and the boundedness of the resolution. Ozair

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et al. [16–19] scrutinized the vector-host prototype of pine wilt disease with vital dynamics to perceive the equilibria and their steadiness by taking into account standard incidence rates, horizontal transference and nonlinear incidence with few dynamic features.

A numerical analysis method developed in early 1990 by German mathematicians Carl Runge [21] and Wilheim Kutta [10] for implicit and explicit iterative methods referred generally as RK45 or simply as the Runge-Kutta method is included in Euler's method. This method is used to solve the initial value problems. A Taylor series expansion shows the consistency of this method. The differential transform method is a numerical procedure for resolving differential equations, initially proposed by Zhou [25]. Hassan [6], Ravi Kanth and Aruna [9], Odibat Zaid et al. [15] and Ümit et al. [22] conferred fundamental definitions of differential transformations, operational properties and illustrated the efficiency of DTM by captivating some numerical examples. Arenas et al. [3] used differential transform procedure and presented an analysed model of circulation of seasonal diseases. Lee et al. [11–14] gave sundry models of pine wilt disease considering non-linear stretching rate and horizontal transmission. Ahmad et al. [2] and Abuasd et al. [1] used contrasting transformation methods and appraised SIS and SI epidemic prototype model of pine wilt disease. Jabberi et al. [8] appraised the model of avian-human influenza epidemic by utilising worthy numerical method. Shah et al. [23] gave semi-analytical resolution to the pine wilt disease by using a duo of domain composition method and Laplace transforms. Rahmann et al. [20] conferred the global firmness of pine wilt disease with convex rate of incidence. Ifeyima [7] employed the differential transform method to reckon the resolution for syphilis disease model.

In this paper, we conferred DTM and MsDTM for the epidemic models [3, 6, 7, 17, 19] of pine wilt disease (PWD) which brings the fine accumulation for the larger domains.

2. Basics of DTM and MsDTM

In this part, preliminary definitions and operational properties of the DTM and MsDTM are discussed. One can refer to [1, 9, 15, 22].

Denote $F(k)$ by the DTM of a function $f(t)$ and define its inverse by

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_0}, \tag{2.1}$$

$$f(t) = \sum_{k=0}^{\infty} F(k)\,(t-t_0)^k. \tag{2.2}$$

The aforementioned equations (2.1) and (2.2), lead to

$$f(t) = \sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_0} = \sum_{k=0}^{\infty}(t-t_0)^k\,F(k).$$

In view of (2.1) and (2.2), we state the required following properties of DTM:

(1) Let $f(t) = g(t) \pm h(t)$. Then $F(k) = G(k) \pm H(k)$.
(2) For any constant c, if $f(t) = c\,g(t)$, then $F(k) = c\,G(k)$.
(3) Let $f(t) = \dfrac{d\,g(t)}{dt}$. Then $F(k) = (k+1)\,G(k+1)$.
(4) Let $f(t) = \dfrac{d^m g(t)}{dt^m}$. Then $F(k) = (k+1)\,(k+2)\cdots(k+m)\,G(k+m)$.
(5) Let $f(t) = 1$. Then $F(k) = \delta(k)$.
(6) Let $f(t) = t^m$. Then $F(k) = \delta(k-m) = \begin{cases} 1, & k = m, \\ 0, & k \neq m. \end{cases}$
(7) Let $f(t) = g(t)\,h(t)$. Then $F(k) = \displaystyle\sum_{m=0}^{k} H(m)\,G(k-m)$.
(8) Let $f(t) = e^{mt}$. Then $F(k) = \dfrac{m^k}{k!}$.

(9) Let $f(t) = (1+t)^m$. Then $F(k) = \dfrac{m(m-1)(m-2)\cdots(m-k+1)}{k!}$.

(10) Let $f(t) = f_1(t)\, f_2(t)\, \cdots\, f_{n-1}\, f_n(t)$. Then

$$F(k) = \sum_{k_{n-1}=0}^{k} \sum_{k_{n-2}=0}^{k_{n-1}} \cdots \sum_{k_2=0}^{k_3} \sum_{k_1=0}^{k_2} F_1(k_1)\, F_2(k_2-k_1)\, \cdots\, F_{n-1}(k_{n-1}-k_{n-2})\, F_n(k-k_{n-1}).$$

If $f(t)$ is in a finite series, then (2.2) becomes

$$f(t) \approx \sum_{k=0}^{N} F(k)\,(t-t_0)^k.$$

Let $f(t, x, x^1, x^2, \ldots, x^{(t)}) = 0$ be the non-linear initial IVP in $[0, T]$ and let its solution be expressed as

$$x(t) = \sum_{k=0}^{N} a_k\, t^k, \quad t \in [0, T]$$

with the initial conditions $x^{(k)}(0) = c_k$ for $k = 0, 1, \ldots, k-1$. Let $[0, T]$ in M subintervals $[t_{m-1}, t_m]$ with $m = 1, 2, \ldots, M$ and step size $h = \frac{T}{M}$ by using the nodes $t_m = mh$. The main idea of MsDTM is applying the DTM to ODE with IVP $f(t, x, x^1, x^2, \ldots, x^{(t)}) = 0$ over an interval $[0, t_1]$. For the initial conditions $x_i^{(k)}(0) = c_k$, an approximate solution is

$$x_1(t) = \sum_{k=0}^{K} a_{1\,k}\, t^k, \quad t \in [0,\ t_1].$$

For $m \geq 2$ and the subintervals $[t_{m-1}, t_m]$ with the initial conditions $x_m^{(k)}\,(t_{m-1}) = x_{m-1}^{(k)}(t_{m-1})$, we apply the DTM to $f(t, x, x^1, x^2, \ldots, x^{(t)}) = 0$, where t_0 in (2.1) is replaced by t_{m-1} and

$$x_m(t) = \sum_{k=0}^{K} a_{mk}(t-t_{m-1})^k, \quad t \in [t_m, t_{m-1}], \quad \text{where} \quad N = KM.$$

At last, the solution $x(t)$ is denoted by

$$x(t) = \begin{cases} x_1(t) : t \in [0, t_1], \\ x_2(t) : t \in [t_1, t_2], \\ \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_M(t) : t \in [t_{M-1}, t_M]. \end{cases},$$

3. Mathematical Formulation

Here, we discuss the epidemiology model developed by Kamal Shah and Manar A. Alqudah [23] for having insight of the transmission of virus and for obtaining some results for the epidemic model under consideration.

$$\frac{dp}{dt} = \alpha - \rho\, p(t)\, f(t)\, (1 + \sigma\, f(t)) - \mu\, p(t), \tag{3.1}$$

$$\frac{dq}{dt} = \rho\, p(t)\, f(t)\, (1 + \sigma\, f(t)) - (\kappa + \mu)\, q(t), \tag{3.2}$$

$$\frac{dr}{dt} = \kappa\, q(t) - \mu\, r(t), \tag{3.3}$$

$$\frac{de}{dt} = \beta - \lambda\, r(t)\, e(t)\, (1 + \eta\, r(t)) - \nu\, e(t), \tag{3.4}$$

$$\frac{df}{dt} = \lambda\, r(t)\, e(t)\, (1 + \eta\, r(t)) - \sigma\, f(t), \tag{3.5}$$

where the class of susceptible pine trees is denoted by $p(t)$; the class of exposed pine trees by $q(t)$; the infective class of pine trees by $r(t)$; the class of susceptible beetles by $e(t)$; the class of infective beetles by $f(t)$. Further, the parameter α represents the new induction enter in trees; β denotes the new induction enter in beetles; μ indicates the death rate of trees; ν denotes the death rate of beetles; λ represents the rate of growth of nematode; σ denotes the infection saturate in trees; η indicates the infection saturate in beetles; ρ represents the contact rate of trees; κ denotes the contact rate of beetles. Parameterized in equations (3.1)–(3.5) and their corresponding values are $\alpha = 0.009041$; $\beta = 0.057142$; $\mu = 0.0000301$; $\nu = 0.011764$; $\lambda = 0.00305$; $\sigma = 0.01$; $\eta = 0.02$; $\rho = 0.00166$ and $\kappa = 0.002691$, where the parameters are α, β, μ, ν, λ, σ, η, ρ, and κ are positive.

4. Applications of Semi-analytical Methods

4.1. **Solution by DTM Approach.** Let $P(k)$, $Q(k)$, $R(k)$, $E(k)$ and $F(k)$ denote the differential transformations of $p(t)$, $q(t)$, $r(t)$, $e(t)$ and $f(t)$, respectively. Taking DTM to system of ODE (3.1) to (3.5), we have

$$\begin{aligned}
P(k+1) =& \frac{1}{(k+1)}\left[\alpha\,\delta[k] - \rho\sum_{m=0}^{k} P(m)\,F(k-m)\right.\\
&\left. -\rho\,\sigma\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} P(k_1)\,F(k_2-k_1)\,F(k-k_2) - \mu\,P(k)\right],\\
Q(k+1) =& \frac{1}{(k+1)}\left[\rho\sum_{m=0}^{k} P(m)\,F(k-m)\right.\\
&\left. +\rho\,\sigma\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} P(k_1)\,F(k_2-k_1)\,F(k-k_2) - (\kappa+\mu)\,Q(k)\right],\\
R(k+1) =& \frac{1}{(k+1)}\left[\kappa\,Q(k) - \mu\,R(k)\right],\\
E(k+1) =& \frac{1}{(k+1)}\left[\beta - \lambda\sum_{m=0}^{k} R(k)\,E(k-m)\right.\\
&\left. -\lambda\,\eta\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} E(k_1)\,R(k_2-k_1)\,R(k-k_2) - \nu\,E(k)\right],\\
F(k+1) =& \frac{1}{(k+1)}\left[\lambda\sum_{m=0}^{k} R(k)\,E(k-m)\right.\\
&\left. -\lambda\,\kappa\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} R(k_1)\,R(k_2-k_1)\,E(k-k_2) - \sigma\,F(k)\right].
\end{aligned}$$

Now, we consider the initial conditions from [23] which reduce to $P(0) = 300$, $Q(0) = 30$, $R(0) = 20$, $E(0) = 65$ and $F(0) = 20$. Further, we take the parameter values from [22] to solve $P(k+1)$, $Q(k+1)$, $R(k+1)$, $E(k+1)$ and $F(k+1)$ in (4.1)–(4.5) up to order 7. We get $P(k)$, $Q(k)$, $R(k)$, $E(k)$ and $F(k)$, respectively. Then the series form of the solution, up to $k = 7$, with $t_0 = 0$, can be written as follows:

$$\begin{aligned}
p(t) =& \sum_{k=0}^{7} P(k)\,t^k = 300 - 11.95198900\,t - 1.614796494\,t^2 + 0.08982166687\,t^3\\
&+ 0.005477132245\,t^4 - 0.0005481796984\,t^5 + 0.0000009887345282\,t^6\\
&+ 0.000001620417954\,t^7,\\
q(t) =& \sum_{k=0}^{7} Q(k)\,t^k = 30 + 11.87036700\,t + 1.598826143\,t^2 - 0.09125565367\,t^3
\end{aligned}$$

$$- 0.005415729212\,t^4 + 0.0005510940742\,t^5 - 0.000001235914841\,t^6$$
$$- 0.000001619941770\,t^7,$$

$$r(t) = \sum_{k=0}^{7} R(k)\,t^k = 20 + 0.08012800000t + 0.01597037288t^2 + 0.001433986814\,t^3$$
$$- 0.00006140303175\,t^4 - 0.000002914375816\,t^5 + 0.0000002471803128\,t^6$$
$$- 0.0000000004761838520\,t^7,$$

$$e(t) = \sum_{k=0}^{7} E(k)\,t^k = 65 - 6.258518000\,t + 0.2897544831\,t^2 - 0.01037501678\,t^3$$
$$+ 0.0002274824352\,t^4 + 0.000005435906236\,t^5 - 0.0000005113560370\,t^6$$
$$+ 0.000000003789609074\,t^7,$$

$$f(t) = \sum_{k=0}^{7} F(k)\,t^k = 20 + 5.315720000\,t - 0.2842089452\,t^2 + 0.01035327088\,t^3$$
$$- 0.0002274184806\,t^4 - 0.000005436056708\,t^5 + 0.0000005113563320\,t^6$$
$$- 0.000000003789609570\,t^7.$$

4.2. **Solution by MsDTM Approach.** Let $P_i(k)$, $Q_i(k)$, $R_i(k)$, $E_i(k)$ and $F_i(k)$ denote the multi-step differential transformations of $p(t)$, $q(t)$, $r(t)$, $e(t)$ and $f(t)$, respectively. Then from (3.1)–(3.5), we have

$$P_i(k+1) = \frac{1}{(k+1)}\Bigg[\alpha\,\delta[k] - \rho\sum_{m=0}^{k} P_i(m)\,F_i(k-m) \\ -\rho\,\sigma\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} P_i(k_1)\,F_i(k_2-k_1)\,F_i(k-k_2) - \mu\,P_i(k)\Bigg] \tag{4.1}$$

$$Q_i(k+1) = \frac{1}{(k+1)}\Bigg[\rho\sum_{m=0}^{k} P_i(m)F_i(k-m) \\ +\rho\,\sigma\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} P_i(k_1)\,F_i(k_2-k_1)\,F_i(k-k_2) - (\kappa+\mu)\,Q_i(k)\Bigg] \tag{4.2}$$

$$R_i(k+1) = \frac{1}{(k+1)}\big[\kappa\,Q_i(k) - \mu\,R_i(k)\big] \tag{4.3}$$

$$E_i(k+1) = \frac{1}{i(k+1)}\Bigg[\beta - \lambda\sum_{m=0}^{k} R_i(k)\,E_i(k-m) \\ -\lambda\,\eta\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} E_i(k_1)\,R_i(k_2-k_1)\,R_i(k-k_2) - \nu\,E_i(k)\Bigg] \tag{4.4}$$

$$F_i(k+1) = \frac{1}{(k+1)}\Bigg[\lambda\sum_{m=0}^{k} R_i(k)\,E_i(k-m) \\ -\lambda\,\eta\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2} R_i(k_1)\,R_i(k_2-k_1)\,E_i(k-k_2) - \sigma\,F_i(k)\Bigg]. \tag{4.5}$$

We consider the initial conditions by $P_i(0) = P_{i-1}(0)$, $Q_i(0) = Q_{i-1}(0)$, $R_i(0) = R_{i-1}(0)$, $E_i(0) = E_{i-1}(0)$, and $F_i(0) = F_{i-1}(0)$, where $P_0(0) = 300$, $Q_0(0) = 30$, $R_0(0) = 20$, $E_0(0) = 65$ and $F_0(0) = 20$.

Further, by applying the parameter values, we solve $P_i(k+1)$, $Q_i(k+1)$, $R_i(k+1)$, $E_i(k+1)$ and $F_i(k+1)$ in (4.1)–(4.5) up to order 7. We get $P_i(k)$, $Q_i(k)$, $R_i(k)$, $E_i(k)$ and $F_i(k)$, respectively.

Then the series form of the solution, up to $k = 7$, can be written as

$$p(t) = \begin{cases} p_0(t) = & 300 - 11.95198900\,t - 1.614796494\,t^2 + 0.08982166687\,t^3 \\ & +0.005477132245\,t^4 - 0.0005481796984\,t^5 + 0.0000009887345282\,t^6 \\ & +0.000001620417954\,t^7, \qquad t \in [0,1]; \\ p_1(t) = & 286.5279667 - 14.89293253\,(t-1) - 1.317903410\,(t-1)^2 \\ & +0.1063213120\,(t-1)^3 + 0.002803273710\,(t-1)^4 \\ & -0.0005118628586\,(t-1)^5 + 0.00001047998275\,(t-1)^6 \\ & +0.000001077555458\,(t-1)^7, \qquad t \in [1,2]; \\ p_2(t) = & 270.4257550 - 17.20105178\,(t-2) - 0.9870606180\,(t-2)^2 \\ & +0.1126591282\,(t-2)^3 + 0.0004339294175\,(t-2)^4 \\ & -0.0004302944512\,(t-2)^5 + 0.00001606760371\,(t-2)^6 \\ & +0.0000005291174994\,(t-2)^7, \qquad t \in [2,3]; \\ p_3(t) = & 252.3503220 - 18.83751178\,(t-3) - 0.6505322030\,(t-3)^2 \\ & +0.1104283382\,(t-3)^3 - 0.001462266873\,(t-3)^4 \\ & -0.0003261396994\,(t-3)^5 + 0.00001812723368\,(t-3)^6 \\ & +0.00000008108773619\,(t-3)^7, \qquad t \in [3,4]; \\ p_4(t) = & 232.9709361 - 19.81466199\,(t-4) - 0.3310098324\,(t-4)^2 \\ & +0.1016807631\,(t-4)^3 - 0.002821293542\,(t-4)^4 \\ & -0.0002180828358\,(t-4)^5 + 0.00001753205548\,(t-4)^6 \\ & -0.0000002273280639\,(t-4)^7, \qquad t \in [4,5]; \end{cases}$$

$$q(t) = \begin{cases} q_0(t) = & 30 + 11.87036700\,t + 1.598826143\,t^2 - 0.09125565367\,t^3 \\ & -0.005415729212\,t^4 + 0.0005510940742\,t^5 - 0.000001235914841\,t^6 \\ & -0.000001619941770\,t^7, \qquad t \in [0,1]; \\ q_1(t) = & 43.37306999 + 14.77532658\,(t-1) + 1.298024978\,(t-1)^2 \\ & -0.1074854410\,(t-1)^3 - 0.002730954120\,(t-1)^4 \\ & +0.0005133322226\,(t-1)^5 - 0.00001071021962\,(t-1)^6 \\ & -0.000001073437153\,(t-1)^7, \qquad t \in [1,2]; \\ q_2(t) = & 59.33670671 + 17.04049185\,(t-2) + 0.9641350530\,(t-2)^2 \\ & -0.1135237274\,(t-2)^3 - 0.0003575498238\,(t-2)^4 \\ & +0.0004304864248\,(t-2)^5 - 0.00001626067784\,(t-2)^6 \\ & -0.0000005228656001\,(t-2)^7, \qquad t \in [2,3]; \\ q_3(t) = & 77.22786604 + 18.62881229\,(t-3) + 0.6254702770\,(t-3)^2 \\ & -0.1109891336\,(t-3)^3 + 0.001536939032\,(t-3)^4 \\ & +0.0003253120692\,(t-3)^5 - 0.00001827313200\,(t-3)^6 \\ & -0.00000007406239479\,(t-3)^7, \qquad t \in [3,4]; \\ q_4(t) = & 96.37300339 + 19.55444998\,(t-4) + 0.3047032362\,(t-4)^2 \\ & -0.1019538180\,(t-4)^3 + 0.002889885028\,(t-4)^4 \\ & +0.0002165270868\,(t-4)^5 - 0.00001762916008\,(t-4)^6 \\ & +0.0000002341056341\,(t-4)^7, \qquad t \in [4,5]; \end{cases}$$

$$r(t)=\begin{cases} r_0(t)= & 20+0.08012800000\,t+0.01597037288\,t^2+0.001433986814\,t^3 \\ & -0.00006140303175\,t^4-0.000002914375816\,t^5+0.0000002471803128\,t^6 \\ & -0.000000004761838520\,t^7, \qquad t\in[0,1]; \\ r_1(t)= & 20.09746830+0.1161119975\,(t-1)+0.01987845443\,(t-1)^2 \\ & +0.001164128958\,(t-1)^3-0.00007231959050\,(t-1)^4 \\ & -0.000001469364143\,(t-1)^5+0.0000002302368732\,(t-1)^6 \\ & -0.000000004118304447\,(t-1)^7, \qquad t\in[1,2]; \\ r_2(t)= & 20.23454932+0.1590660179\,(t-2)+0.02292558784\,(t-2)^2 \\ & +0.0008645991227\,(t-2)^3-0.00007637959370\,(t-2)^4 \\ & -0.0000001919735100\,(t-2)^5+0.0000001930741245\,(t-2)^6 \\ & -0.000000006251899371\,(t-2)^7, \qquad t\in[2,3]; \\ r_3(t)= & 20.41732913+0.2072056259\,(t-3)+0.02506194849\,(t-3)^2 \\ & +0.0005607953833\,(t-3)^3-0.00007467215960\,(t-3)^4 \\ & +0.0000008276301134\,(t-3)^5+0.0000001458983111\,(t-3)^6 \\ & -0.000000007025341393\,(t-3)^7, \qquad t\in[3,4]; \\ r_4(t)= & 20.65008380+0.2587181846\,(t-4)+0.02630661874\,(t-4)^2 \\ & +0.0002730548598\,(t-4)^3-0.00006859148580\,(t-4)^4 \\ & +0.000001555749043\,(t-4)^5+0.00000009710459377\,(t-4)^6 \\ & -0.000000006777570376\,(t-4)^7, \qquad t\in[4,5]; \end{cases}$$

$$e(t)=\begin{cases} e_0(t)= & 65-6.258518000\,t+0.2897544831\,t^2-0.01037501678\,t^3 \\ & +0.0002274824352\,t^4+0.000005435906236\,t^5-0.0000005113560370\,t^6 \\ & +0.000000003789609074\,t, \qquad t\in[0,1]; \\ e_1(t)= & 59.02109387-5.709200011\,(t-1)+0.2600411213\,(t-1)^2 \\ & -0.009420772307\,(t-1)^3+0.0002471858814\,(t-1)^4 \\ & +0.000002495362180\,(t-1)^5-0.0000004618832888\,(t-1)^6 \\ & +0.000000009749984834\,(t-1)^7, \qquad t\in[1,2]; \\ e_2(t)= & 53.56276345-5.216381568\,(t-2)+0.2332801634\,(t-2)^2 \\ & -0.008415945610\,(t-2)^3+0.0002531073002\,(t-2)^4 \\ & -0.00000004692800220\,(t-2)^5-0.0000003823409543\,(t-2)^6 \\ & +0.00000001251322025\,(t-2)^7, \qquad t\in[2,3]; \\ e_3(t)= & 48.57149879-4.774059086\,(t-3)+0.2095450324\,(t-3)^2 \\ & -0.007411186907\,(t-3)^3+0.0002475844465\,(t-3)^4 \\ & -0.000002071743618\,(t-3)^5-0.0000002920858472\,(t-3)^6 \\ & +0.00000001296706882\,(t-3)^7, \qquad t\in[3,4]; \\ e_4(t)= & 43.99981878-4.376224265\,(t-4)+0.1887721500\,(t-4)^2 \\ & -0.006446958677\,(t-4)^3+0.0002332926454\,(t-4)^4 \\ & -0.000003556862632\,(t-4)^5-0.0000002040897123\,(t-4)^6 \\ & +0.00000001200549015\,(t-4)^7, \qquad t\in[4,5]; \end{cases}$$

$$
f(t)=\begin{cases}
f_0(t)= & 20+5.315720000\,t-0.2842089452\,t^2+0.01035327088\,t^3\\
& -0.0002274184806\,t^4-0.000005436056708\,t^5+0.0000005113563320\,t^6\\
& -0.000000003789609570\,t^7, \qquad t\in[0,1];\\
f_1(t)= & 25.04163197+4.777428104\,(t-1)-0.2545604390\,(t-1)^2\\
& +0.009399280723\,(t-1)^3-0.0002471226746\,(t-1)^4\\
& -0.000002495510894\,(t-1)^5+0.0000004618835803\,(t-1)^6\\
& -0.000000009749985324\,(t-1)^7, \qquad t\in[1,2];\\
f_2(t)= & 29.57364975+4.295506803\,(t-2)-0.2278635780\,(t-2)^2\\
& +0.008394705373\,(t-2)^3-0.0002530448328\,(t-2)^4\\
& +0.00000004678102880\,(t-2)^5+0.0000003823412425\,(t-2)^6\\
& -0.00000001251322074\,(t-2)^7, \qquad t\in[2,3];\\
f_3(t)= & 33.64943506+3.863954020\,(t-3)-0.2041917944\,(t-3)^2\\
& +0.007390195077\,(t-3)^3-0.0002475227095\,(t-3)^4\\
& +0.000002071598364\,(t-3)^5+0.0000002920861320\,(t-3)^6\\
& -0.00000001296706930\,(t-3)^7, \qquad t\in[3,4];\\
f_4(t)= & 37.31634232+3.476762946\,(t-4)-0.1834815185\,(t-4)^2\\
& +0.006426212347\,(t-4)^3-0.0002332316304\,(t-4)^4\\
& +0.000003556719076\,(t-4)^5+0.0000002040899938\,(t-4)^6\\
& -0.00000001200549062\,(t-4)^7, \qquad t\in[4,5].
\end{cases}
$$

5. Discussion

In this proposed work, we wish to create acognizance on DTM and consistent revision of DTM, that is, Ms-DTM provides better convergence of series solutions. To obtain numerical solutions for the outlined equations of epidemic model of pine wilt disease, many scholars used the RK 4 method. But here we show that DTM and Ms-DTM methods give better converging solutions than the RK 4 method. Both linear and non-linear models can be solved by using these methods which give instant observable figurative terms of analytical and numerical approximate solutions to both types of linear and non-linear models containing differential coefficients. In this work we gave a convergent series solution for epidemic model of pine wilt disease, to solve these equations, we used DTM and Ms-DTM methods without considering any restrictive norms.

These procedures provide a noteworthy performance after plotting graphs. Figures 1, 2, 3 show an instance of change in approximate and numerical solutions of susceptible (p(t)), exposed $(q(t))$ and infected $(r(t))$ trees with respect to time t. Figures 4, 5 expose the solution for susceptible $(e(t))$ and infected $(f(t))$ beetles at time t and these figures conclude that both multi-step DTM and RK 4 methods give approximately, but not exactly, the same convergence in numerical solutions. On spotting these, we propose that these methods are more effectual and accurate than the RK 4 method, and these can be used to predict investigative solutions for non-linear system of DE s. Here, we observed that by employing differential transformation technique, the result attained has a small interval of convergence, but by employing multi-step differential transformation technique, an extensive interval of convergence for a series solution was attained. The accuracy of the numerical approximation can be enhanced by adopting inclination in K and declination in step size h.

On observing the above fact, we emphasize that this algorithm is much precise and effectual technique when related to RK 4 technique. This technique works efficiently in managing systems of equations with differential coefficients directly to extensive interval of convergence for the series solution with minimum size of computations and this shows that Ms-DTM is more promising and reliable in solving linear and non-linear models when compared with a currently existing technique.

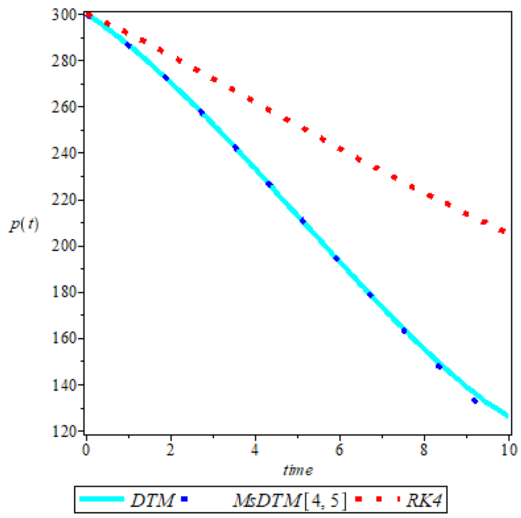

FIGURE 1. Numerical solutions for susceptible $p(t)$ pine trees in a time t.

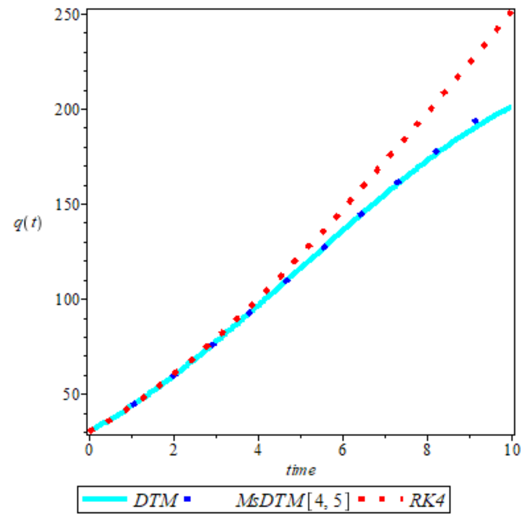

FIGURE 2. Numerical solutions for exposed $q(t)$ pine trees in a time t.

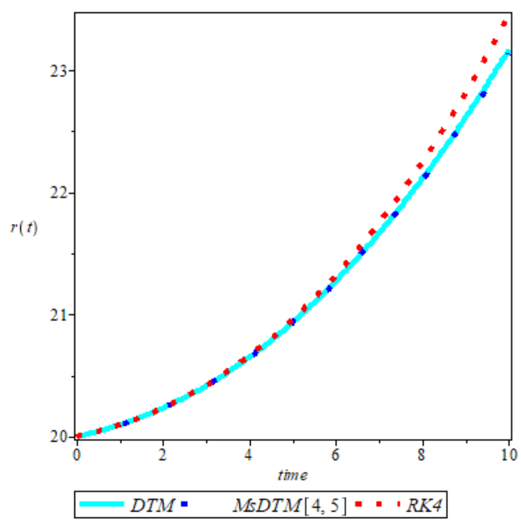

FIGURE 3. Numerical solutions for infected $r(t)$ pine trees in a time t.

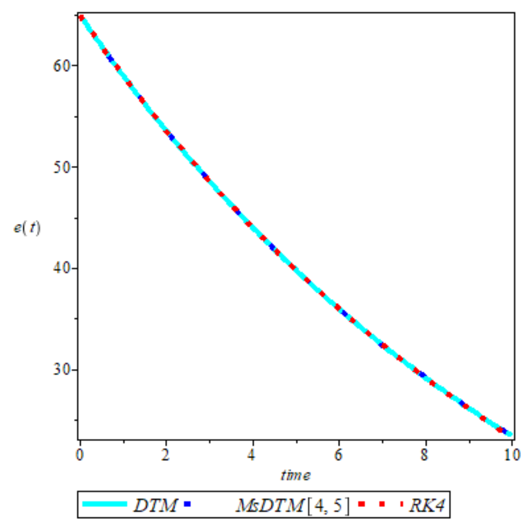

FIGURE 4. Numerical solutions for susceptible $e(t)$ beetles in a time t.

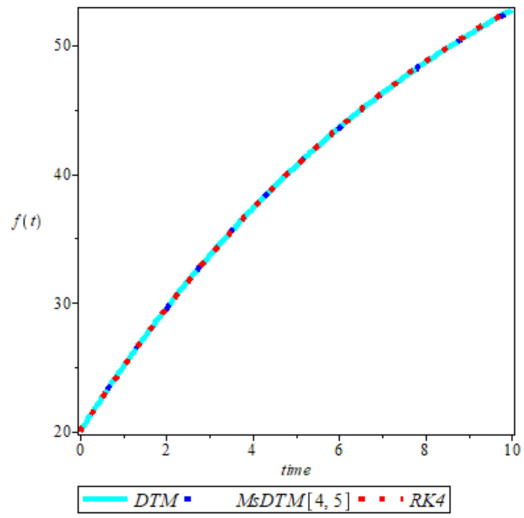

Figure 5. Numerical solutions for infected $f(t)$ beetles in a time t.

6. Conclusion

In this paper, we started with the definitions and properties of DTM and MsDTM, then DTM and MsDTM are applied to solve the system of equations characterized by the epidemic model of pine wilt disease. The results obtained by DTM and MsDTM are compared graphically. We show that by using Ms-DTM we obtain more promising and reliable results in solving linear and non-linear models as compared with currently existing technique.

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