This volume is dedicated to the memory of excellent mathematician, teacher and person Edem Lagvilava

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THE MATHEMATICAL LEGACY OF EDEM LAGVILAVA

Edem Lagvilava was a person, who became a legend in his lifetime. Although each of his phrases, gestures, or facial expressions exerted a tremendous influence on each of us, he remained almost invisible to bystanders due to his humility and his delicate attitude. The legends about his first steps in mathematics are being passed down to the generations following in his footsteps and it may take many generations to fully comprehend the scope and value of his contribution to the field of mathematics.

After graduating with honors from the Mathematics Department of Lomonosov Moscow State University in 1972, he was faced with the task of choosing a topic for his doctoral thesis, but this choice was a prerogative of his scientific advisor Gigla Janashia. After consulting with Edem, Janashia selected the following topic: The creation of a method for approximate computation of factorization of positive definite matrix functions. Although Georgia had a long tradition in conducting researches in the factorization of matrix functions, the foundations of which were laid by Niko Muskhelishvili, the posed problem was nevertheless extremely challenging.

When a scientific adviser sets a problem for his exceptionally talented student, even if this problem is a conjecture formulated by a world-renowned scientist, the student's hard work may determine the course of events in three possible directions: (i) He/she proves that the conjecture is true, (ii) he/she constructs a counterexample, showing that the conjecture is not valid, and (iii) he/she partially proves the conjecture in a specific, albeit important case. In all of the aforementioned three cases, the merits and the scientific contribution of the student do not disappear without leaving a trace, but will be appreciated in the community of mathematicians. The objective of the topic selected by Gigla Janashia and Edem Lagvilava went beyond the natural frames described above. The existence of the factorization of positive definite matrix functions was proven by Norbert Wiener. Given the high potential of using his theories in practice, Wiener was extensively studying the method of approximate computation of factorization. In 1958, he published an algorithm for factorization under the certain restrictions on matrix functions. Edem Lagvilava's objective was to find a better algorithm. However, his contribution to resolving the problem could not be assessed by using the three aforementioned criteria. There was only one way of creating a better algorithm, since otherwise, Edem's efforts would be wasted. The risk was high from the very start, because there were no guarantees that a better algorithm existed, particularly if we take into account the history of this problem and the prominence of Wiener as a scientist. Logically, there were no guarantees. Only deep intuition could have prompted both Gigla and Edem that it was worth to put effort into it.

The action plan from the very beginning consisted in the following: To impose no additional restrictions on the matrix function, except the necessary and sufficient condition for the existence of factorization. In other words, the algorithm should be created under the most general conditions. Edem found the main difficulty very quickly and began a slow but consistent work of unfastening this complicated knot. He did not use ready theories developed by others, personally creating everything that was necessary to make an algorithm. It turned out years later that the main instrument that Edem created and used to achieve his goal was the full classification of wavelet matrices and multi-rate filter banks. A similar theory of such filters, significant for engineering devices, was developed in a different form much later and it is now a part of classical literature. Gigla Janashia's habitual style of publishing all his works in a complete form was a major impetus for Edem Lagvilava not to stop halfway and to see what he had started through to the end: he described the method for approximate computation; proved the convergence of the method to the solution; and estimated the rate of convergence. For different reasons, these results were published in the leading engineering and mathematical journals only in 2011–2014, although Edem had achieved them (jointly with Gigla Janashia) 30–40 years ago.

Ultimately, the scientific value of Edem Lagvilava's end results was beyond any traditional methods for evaluation: such as publications, reviews, and citations. His algorithm was tested in a project financed with a large-scale military grant at the University of Maryland (2008–2011). Neither were his merits acknowledged in any of the traditional ways such as granted titles, prizes, or high positions. The method he developed, known today as the Janashia-Lagvilava method, was patented in the USA

in 2016 which is in fact unprecedented in the field of mathematics. This method is already being applied in the field of neuroscience and is expected to be used on a broader scale in engineering, economics, and so forth.

In this volume dedicated to Edem Lagvilava, we attempted to publish the papers of the scientists, who are closely familiar with and highly appreciate the Janashia–Lagvilava method.

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