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DETERMINATION OF THE INFLUENCE OF FLUID WITHDRAWAL FROM THE TRANSPORT LINE AND CONNECTIONS TO IT ON THE HYDRODYNAMICS OF FLUID MOTION IN THE RESERVOIR-PIPELINE SYSTEM

NURLANA A. AGAYEVA

Abstract. A hydrodynamic model of the process of fluid motion in the reservoir-pipeline system in the case of fluid withdrawal and connections to it was constructed and the related equations were solved. An analytic expression was obtained to determine the influence of connections and fluid withdrawal from the transport line on the dynamics of pressure in the bottom hole and reservoir productivity

1. Introduction

In practice, the cases of connections or fluid withdrawals from the existing oil transportation line are not uncommon. They show significant influence on the already steady state of a well, get them out of this state and negatively affect the productivity of the reservoir. To determine this transitional mode on the well operation, it is necessary to consider the process of fluid motion in the reservoir-pipeline system. So far, little attention has been paid to this issue, and therefore it has remained little studied.

So, for example, in [1], the fluid motion in the reservoir-well system was considered, and the influence of the fluid motion in the transport line on the hydrodynamics of a fluid flow in this system was given as the influence of pressure on the wellhead determined experimentally.

The influence of connections and fluid withdrawal from a separately taken line on the hydrodynamics of the fluid flow in this line was studied in [7].

The problem of construction of integral mathematical models of filtration of fluid in reservoirs and the flow of gas-fluid mixtures in oil gathering pipeline networks for the stationary case of their motion was considered in [2].

The work [10] is devoted to the physical and mathematical formalization of the development and software implementation of computational algorithms for modeling non-stationary three-phase flows in the conjugated reservoir-well– ESP system. Therefore, modeling and study of the influence of the fluid withdrawal from the transport line and connections to it on the hydrodynamics of fluid motion in the reservoir-pipeline system have great theoretical and practical importance.

2. Problem Statement and Methods for Solving it

The fluid motion occurs in the conjugated reservoir – pipeline system of the transport line.

2.1. Fluid filtration.

First, we consider flat-radial filtration of homogeneous fluid in a uniform reservoir (Figure 1).

Here, the filtration equation is of the form [9]

$$\frac{\partial^2 \Delta P}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta P}{\partial r} = \frac{1}{\chi}\frac{\partial \Delta P}{\partial t}, \qquad r_c \le r \le R_k; \;\; t > 0, \tag{2.1}$$

where $\Delta P = P_k - P$; $\chi = \frac{k}{\mu\beta^*}$.

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The initial and boundary conditions

$$\Delta P|_{r=R_k} = 0, \qquad t > 0, \tag{2.2}$$

$$\Delta P|_{r=r_c} = P_k - P_c(t), \qquad t > 0. \tag{2.3}$$

The solution of equation (2.1) for the boundary condition has the form [9]

$$\Delta P = \frac{\ln\left(\frac{R_k}{r}\right)}{\ln\left(\frac{R_k}{r_c}\right)}\Delta P_{cy} - \pi\Delta P_{c_1}\sum_{\nu=1}^{\infty}\frac{J_0\left(x_\nu \frac{R_k}{r_c}\right)J_0\left(x_\nu\right)}{J_0^2\left(x_\nu \frac{R_k}{r_c}\right) - J_0^2\left(x_\nu\right)}\Big[J_0\Big(x_\nu \frac{r}{r_c}\Big)Y_0\Big(x_\nu \frac{R_k}{r_c}\Big)$$

$$-Y_0\Big(x_\nu \frac{r}{r_c}\Big)J_0\Big(x_\nu \frac{R_k}{r_c}\Big)\Big] \exp\Big(-\frac{x_\nu^2\,\chi\,t}{r_c^2}\Big). \tag{2.4}$$

$$\Delta P_{c_1} = P_c(0) - P_{c_1}, \qquad \Delta P_{cy} = P_k - P_c(0), \tag{2.5}$$

where P_{c_1} is the fixed pressure in the bottom hole after its change.
Then from formula (2.5), allowing for the expression (2.4), we get

$$\Delta P_0 = \frac{\ln\left(\frac{R_k}{r}\right)}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - \pi\sum_{\nu=1}^{\infty}A(x_\nu)U\Big(x_\nu \frac{r}{r_c}\Big)\exp\Big(-\frac{x_\nu^2\,\chi\,t}{r_c^2}\Big). \tag{2.6}$$

$$\Delta P_0 = \frac{\Delta P}{\Delta P_{c_1}},$$

where x_ν are the roots of the transcendental equation

$$Y_0\left(x_\nu\right)J_0\Big(x_\nu \frac{R_k}{r_c}\Big) - J_0\left(x_\nu\right)Y_0\Big(x_\nu \frac{R_k}{r_c}\Big) = 0,$$

$$A\Big(x_\nu \frac{R_k}{r_c}\Big) = \frac{J_0\left(x_\nu \frac{R_k}{r_c}\right)J_0\left(x_\nu\right)}{J_0^2\left(x_\nu \frac{R_k}{r_c}\right) - J_0^2\left(x_\nu\right)}, \tag{2.7}$$

$$U\Big(x_\nu \frac{r}{r_c}\Big) = \Big[J_0\Big(x_\nu \frac{r}{r_c}\Big)Y_0\Big(x_\nu \frac{R_k}{r_c}\Big) - Y_0\Big(x_\nu \frac{r}{r_c}\Big)J_0\Big(x_\nu \frac{R_k}{r_c}\Big)\Big]. \tag{2.8}$$

The solution of equation (2.1) under the boundary condition (2.3) may be implemented by the method of the Duhamel integral [11]

$$\Delta P(r,t) = f(0)P_0(t) + \int_0^t \dot f(\tau)\Delta P_0[r;(t-\tau)]d\tau, \tag{2.9}$$

$$f(t) = P_k - P_c(t). \tag{2.10}$$

Then from expression (2.9), allowing for formulas (2.6) and (2.10), we get

$$\Delta P(r,t) = (P_k - P_c(0))\Big(\frac{\ln\left(\frac{R_k}{r}\right)}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - \pi\sum_{\nu=1}^{\infty}A(x_\nu)U\Big(x_\nu \frac{r}{r_c}\Big)\exp\Big(-\frac{x_\nu^2\,\chi\,t}{r_c^2}\Big)\Big)$$

$$-\int_0^t \dot P_c(\tau)\pi\sum_{\nu=1}^{\infty}\Big(\frac{\ln\left(\frac{R_k}{r}\right)}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - \pi\sum_{\nu=1}^{\infty}A(x_\nu)U\Big(x_\nu \frac{r}{r_c}\Big)\Big)\exp\Big(-\frac{x_\nu^2\,\chi(t-\tau)}{r_c^2}\Big)\,d\tau. \tag{2.11}$$

The fluid rate at the moment t through the lateral surface of the well of radius r_c is determined by the formula

$$Q|_{r=r_c} = -2\pi r_c h\frac{k}{\mu}\left.\frac{\partial\Delta P}{\partial r}\right|_{r=r_c}. \tag{2.12}$$

Substituting expression (2.11) in formula (2.12), we get

$$Q|_{r=r_c} = 2\pi h\frac{k}{\mu}(P_k - P_c(0))\left(\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - 2B(x_\nu)\exp\left(-b_\nu t\right)\right)$$

$$-2\pi h\frac{k}{\mu}\int\limits_0^t \dot{P}_c(\tau)\left(\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}} - 2B(x_\nu)\exp\left(-b_\nu t\right)\right)d\tau, \tag{2.13}$$

where

$$B\left(x_\nu\right) = \frac{J_0^2\left(x_\nu\frac{R_k}{r_c}\right)}{J_0^2\left(x_\nu\right) - J_0^2\left(x_\nu\frac{R_k}{r_c}\right)}, \quad b_\nu = \left(\frac{x_\nu^2\,\chi}{r_c^2}\right). \tag{2.14}$$

Applying the Laplace transform, with regard to the convolution theorem, from expression (2.13), we get

$$\bar{Q}|_{r=r_c} = 2\pi h\frac{k}{\mu}(P_k - P_c(0))\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}}\frac{1}{s} - 4\pi h\frac{k}{\mu}B(x_\nu)\frac{(P_k - P_c(0))}{s+b_\nu}$$

$$-2\pi h\frac{k}{\mu}\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}}\left(\bar{P}_c - \frac{P_c(0)}{s}\right) + 4\pi h\frac{k}{\mu}B(x_\nu)\left(\frac{s\bar{P}_c}{s+b_\nu} - \frac{P_c(0)}{s+b_\nu}\right). \tag{2.15}$$

2.2. **The motion of fluid through a lifting pipes column.** We now consider the fluid motion in the lifting pipes column. Accepting fluid as capillary compressible homogeneous for the equation of fluid motion in the pipe and the continuity equation, we have [4, 6, 8]

$$\begin{gathered} -\frac{\partial P}{\partial x} = \frac{\partial Q_1}{\partial t} + 2aQ_1, \\ -\frac{1}{c^2}\frac{\partial P}{\partial t} = \frac{\partial Q_1}{\partial x}, \end{gathered} \tag{2.16}$$

where $c^2 = \frac{\partial P}{\partial \rho}$; c is a speed of sound in the fluid, $Q_1 = \rho u$ is mass fluid flow rate per unit area of pipe, ρ is fluid density, u is fluid flow rate averaged in the cross-section of the pipe, a is a drag coefficient per unit area of its cross-section.

Differentiating both sides of the first equation at time t and the second equation with respect to the equation with respect to x and subtracting them term by term, we get

$$\frac{\partial^2 Q_1}{\partial t^2} = c^2\frac{\partial^2 Q_1}{\partial x^2} - 2a\frac{\partial Q_1}{\partial t}. \tag{2.17}$$

Expression (2.17) can be represented as

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} - 2a\frac{\partial u}{\partial t}. \tag{2.18}$$

We consider the fluid motion in the pipe as the sum of two motions: the transportable motion of a fluid column u_e and the relative motion of the cross sections of a fluid column u_r from its compressibility

$$u = u_e + u_r, \tag{2.19}$$

Substituting expressions (2.19) in equation (2.18), by virtue of its linearity, we get

$$\frac{d^2 u_e}{dt^2} + 2a\frac{du_e}{dt} = \frac{\dot{P}_c - \dot{P}_{\text{y}}}{l\rho}, \tag{2.20}$$

$$\frac{\partial^2 u_r}{\partial t^2} = c^2\frac{\partial^2 u_r}{\partial x^2} - 2a\frac{\partial u_r}{\partial t} + \frac{\dot{P}_y - \dot{P}_{\text{c}}}{l\rho}. \tag{2.21}$$

The initial and boundary conditions are represented as follows:

$$u_e|_{t=0} = \frac{G_0}{f}, \qquad 0 < x < l, \tag{2.22}$$

$$\left.\frac{du_e}{dt}\right|_{t=0} = 0, \qquad 0 < x < l, \tag{2.23}$$

$$u_r|_{t=0} = 0, \qquad 0 < x < l, \tag{2.24}$$

$$\left.\frac{\partial u_r}{\partial t}\right|_{t=0} = 0, \qquad 0 < x < l, \tag{2.25}$$

$$\left.\frac{\partial u_r}{\partial x}\right|_{x=l} = 0, \qquad t > 0, \tag{2.26}$$

$$u_r|_{x=0} = 0, \qquad t > 0, \tag{2.27}$$

$$fu|_{x=0} = -\frac{2\pi k}{\mu} r_c h \frac{\partial \Delta P}{\partial r}|_{r=r_c}, \qquad t > 0. \tag{2.28}$$

Applying the Laplace transform and taking into account convolution theorems, allowing for the initial conditions (2.22) and (2.23), from equation (2.20), we get

$$u_e(t) = \frac{G_0}{f} + \frac{1}{l\rho}\int_0^t P_c(\tau)\, e^{-2a(t-\tau)} d\tau - \frac{1}{l\rho}\int_0^t P_y(\tau)\, e^{-2a(t-\tau)} d\tau$$
$$+ \frac{P_y(0) - P_{c1}(0)}{l\rho}\Big(\frac{1}{2a} - \frac{1}{2a} e^{-2at}\Big). \tag{2.29}$$

Allowing for the initial and boundary conditions (2.24) and (2.27), we look for a solution of equation (2.21) in the form

$$u_r = \sum_{i=1}^{n} \varphi_i(t)\Big(1 - \cos\frac{i\pi x}{l}\Big), \tag{2.30}$$

where $\varphi_i(t)$ is an unknown function depending on time t and l is the length of the pipe. Substituting expression (2.30) in equation (2.21), multiplying both sides of the obtained expression by $\big(1 - \cos\frac{i\pi x}{l}\big)$ and integrating it from 0 to l, we get the following equation:

$$\ddot{\varphi}_i(t) + 2a\dot{\varphi}_i(t) + \frac{c^2\pi^2 i^2}{3l^2}\varphi_i(t) - \frac{2}{3\rho l}\dot{P}_y(t) + \frac{2}{3\rho l}\dot{P}_c(t) = 0. \tag{2.31}$$

Applying the Laplace transform, convolution and conversion theorems with regard to the initial conditions (2.24) and (2.25), from equation (2.31), we get

$$\varphi_i = -\frac{2a_{11}}{3l\rho b_{11}}\Big[\int_0^t P_y(\tau)\, e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau)) d\tau$$
$$- \int_0^t P_c(\tau)\, e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau)) d\tau\Big] + \frac{2}{3l\rho}\Big[\int_0^t P_y(\tau)\, e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau)) d\tau$$
$$- \int_0^t P_c(\tau)\, e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau)) d\tau\Big] + \frac{2(P_c(0) - P_y(0))}{3l\rho}\Big(\frac{e^{-a_{11}t}\sin(b_{11}t)}{b_{11}}\Big), \tag{2.32}$$

where a_{11} and b_{11} are real and imaginary parts of the root of the equation

$$s^2 + 2as + \frac{c^2\pi^2 i^2}{l^2} = 0. \tag{2.33}$$

Substituting expression (2.32) in formula (2.30), we get

$$u_r = \sum_{i=1}^{n} \Big(-\frac{2a_{11}}{3l\rho b_{11}} \Big[\int_0^t P_y(\tau)\, e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau))d\tau - \int_0^t P_c(\tau)\, e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau))d\tau \Big] + \frac{2}{3l\rho} \Big[\int_0^t P_y(\tau)\, e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau))d\tau - \int_0^t P_c(\tau)\, e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau))d\tau \Big] + \frac{2(P_c(0) - P_y(0))}{3l\rho} \Big(\frac{e^{-a_{11}t}\sin(b_{11}t)}{b_{11}} \Big) \Big) \Big(1 - \cos\frac{i\pi x}{l}\Big). \tag{2.34}$$

The fluid flow rate

$$Q_1 = fu = f(u_e + u_r). \tag{2.35}$$

Substituting expressions (2.29) and (2.34) in formula (2.35), we get the fluid flow rate in the vertical pipe,

$$Q_1 = f\Big(\frac{G_0}{f} + \frac{1}{l\rho}\int_0^t P_c(\tau)\, e^{-2a(t-\tau)}d\tau - \frac{1}{l\rho}\int_0^t P_y(\tau)\, e^{-2a(t-\tau)}d\tau + \frac{P_y(0) - P_c(0)}{l\rho}\Big(\frac{1}{2a} - \frac{1}{2a}e^{-2at}\Big) + \sum_{i=1}^{n} \Big(-\frac{2a_{11}}{3l\rho b_{11}} \Big[\int_0^t P_y(\tau)\, e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau))d\tau - \int_0^t P_c(\tau)\, e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau))d\tau \Big] + \frac{2}{3l\rho} \Big[\int_0^t P_y(\tau)\, e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau))d\tau - \int_0^t P_c(\tau)\, e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau))d\tau \Big] + \frac{2(P_c(0) - P_y(0))}{3l\rho} \Big(\frac{e^{-a_{11}t}\sin(b_{11}t)}{b_{11}} \Big) \Big) \Big(1 - \cos\frac{i\pi x}{l}\Big)\Big). \tag{2.36}$$

Applying the Laplace transform, from expression (2.36), we get

$$\overline{Q}_1 = f\Big[\frac{G_0}{fs} + \frac{1}{\rho l}\frac{\overline{P}_c}{s+2a} - \frac{1}{\rho l}\frac{\overline{P}_y}{s+2a} + \frac{1}{2al}\frac{(P_y(0) - P_c(0))}{\rho s} - \frac{1}{2al}\frac{(P_y(0) - P_c(0))}{\rho(s+2a)} + \sum_{i=1}^{n}\Big(1 - \cos\frac{i\pi x}{l}\Big)\Big(\frac{-2a_{11}}{3l\rho}\frac{\overline{P}_y}{(s+a_{11})^2 + b_{11}^2} + \frac{2}{3l\rho}\frac{\overline{P}_y(s+a_{11})}{(s+a_{11})^2 + b_{11}^2} - \frac{2}{3l\rho}\frac{\overline{P}_c(s+a_{11})}{(s+a_{11})^2 + b_{11}^2} + \frac{2a_{11}}{3l\rho}\frac{\overline{P}_c}{(s+a_{11})^2 + b_{11}^2} + \frac{1}{3l\rho}\frac{(P_c(0) - P_y(0))}{(s+a_{11})^2 + b_{11}^2} \Big)\Big]. \tag{2.37}$$

Based on the continuity condition (2.28),

$$Q|_{r=r_c} = Q_1|_{x=0}\,. \tag{2.38}$$

Substituting expressions (2.15) and (2.37) in formula (2.38) with regard to only one term of the series in the first approximation, we get the following Volterra type integral equation from which we determine $P_c(t)$,

$$2\pi h\frac{k}{\mu}(P_k - P_c(0))\frac{1}{\ln\Big(\frac{R_k}{r_c}\Big)}\frac{\Delta P_{cy}}{\Delta P_{c_1}}\frac{1}{s} - 4\pi h\frac{k}{\mu}B(x_\nu)\frac{(P_k - P_c(0))}{s + b_\nu}$$

$$-2\pi h\frac{k}{\mu}\frac{1}{\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c_1}}\left(\bar{P}_c-\frac{P_c(0)}{s}\right)+4\pi h\frac{k}{\mu}B(x_\nu)\left(\frac{s\bar{P}_c}{s+b_\nu}-\frac{P_c(0)}{s+b_\nu}\right)$$
$$=f\Bigg[\frac{G_0}{fs}+\frac{1}{\rho l}\frac{\overline{P}_c}{s+2a}-\frac{1}{\rho l}\frac{\overline{P}_y}{s+2a}+\frac{1}{2al}\frac{(P_y(0)-P_c(0))}{\rho s}-$$
$$-\frac{1}{2al}\frac{(P_y(0)-P_c(0))}{\rho(s+2a)}+\sum_{i=1}^{n}\left(1-\cos\frac{i\pi x}{l}\right)\left(\frac{-2a_{11}}{3l\rho}\frac{\overline{P}_y}{(s+a_{11})^2+b_{11}^2}+\frac{2}{3l\rho}\frac{\overline{P}_y(s+a_{11})}{(s+a_{11})^2+b_{11}^2}\right.$$
$$\left.-\frac{2}{3l\rho}\frac{\overline{P}_c(s+a_{11})}{(s+a_{11})^2+b_{11}^2}+\frac{2a_{11}}{3l\rho}\frac{\overline{P}_c}{(s+a_{11})^2+b_{11}^2}+\frac{1}{3l\rho}\frac{(P_c(0)-P_y(0))}{(s+a_{11})^2+b_{11}^2}\right)\Bigg]. \tag{2.39}$$

Applying the Laplace transform and taking into account the conversion and convolution theorems [3, 5], from equation (2.39), we have

$$P_c(t)=\frac{1}{A_3\eta_1\eta_2(\eta_1-\eta_2)}\left(\rho lG_0(2b_\nu a(\eta_1-\eta_2)+e^{\eta_1 t}\eta_2(\eta_1+b_\nu)(\eta_1+2a)\right.$$
$$\left.-e^{\eta_2 t}\eta_1(\eta_2+b_\nu)(\eta_2+2a))\right)-\frac{f}{A_3(\eta_1-\eta_2)}\left(\int_0^t P_y(\tau)e^{\eta_1(t-\tau)}d\tau(\eta_1+b_\nu)\right.$$
$$\left.-\int_0^t P_y(\tau)e^{\eta_2(t-\tau)}d\tau)(\eta_2+b_\nu\right)+\frac{fP_y(0)}{2aA_3}\frac{1}{\eta_1\eta_2(\eta_1-\eta_2)}(2b_\nu a(\eta_1-\eta_2)+e^{\eta_1 t}\eta_2(\eta_1+b_\nu)(\eta_1+2a)$$
$$-e^{\eta_2 t}\eta_1(\eta_2+b_\nu)(\eta_2+2a))-\frac{fP_c(0)}{2aA_3}\frac{1}{\eta_1\eta_2(\eta_1-\eta_2)}(2b_\nu a(\eta_1-\eta_2)+e^{\eta_1 t}\eta_2(\eta_1+b_\nu)(\eta_1+2a)$$
$$-e^{\eta_2 t}\eta_1(\eta_2+b_\nu)(\eta_2+2a))-\frac{fP_y(0)}{2aA_3}\frac{1}{(\eta_1-\eta_2)}(-e^{\eta_1 t}(\eta_1+b_\nu)-e^{\eta_2 t}(\eta_2+b_\nu))$$
$$+\frac{fP_c(0)}{2aA_3}\frac{1}{(\eta_1-\eta_2)}(-e^{\eta_1 t}(\eta_1+b_\nu)-e^{\eta_2 t}(\eta_2+b_\nu))$$
$$-\frac{A_2\rho lP_k}{A_3}\frac{1}{\eta_1\eta_2(\eta_1-\eta_2)}(2b_\nu a(\eta_1-\eta_2)+e^{\eta_1 t}\eta_2(\eta_1+b_\nu)(\eta_1+2a)$$
$$-e^{\eta_2 t}\eta_1(\eta_2+b_\nu)(\eta_2+2a))+\frac{A_1\rho lP_k}{A_3}\frac{1}{(\eta_1-\eta_2)}(e^{\eta_1 t}(\eta_1+2a)-e^{\eta_2 t}(\eta_2+2a)), \tag{2.40}$$

where $A_1=\frac{4\pi khB(x_\nu)}{\mu}$, $A_2=\frac{2\pi kh}{\mu\ln\left(\frac{R_k}{r_c}\right)}\frac{\Delta P_{cy}}{\Delta P_{c1}}$, $A_3=\rho l(A_1-A_2)$,

$$A_4=\rho l(2aA_1-2aA_2-A_2b_\nu)-f,\qquad A_5=2aA_2\rho lb_\nu+b_\nu f,$$

η_1, η_2 are the roots of the equation $s^2+\frac{A_4}{A_3}s+\frac{A_5}{A_3}=0$.

2.3. **The fluid motion in a trunk pipeline.** Let at some moment of time, a pipeline with flow G be connected to the trunk oil line and fluid with flow G_1 be withdrawn. Then the fluid motion in the trunk pipeline has the form

$$\frac{\partial^2 P}{\partial t^2}=c^2\frac{\partial^2 P}{\partial x^2}-2a_1\frac{\partial P}{\partial t}-\frac{2a_1c^2G}{f_1}\delta(x_1-l_2)+\frac{2a_1c^2G_1}{f_1}\delta(x_1-l_3), \tag{2.41}$$

where G and G_1 are the mass flow rates in the inlet and outlet pipes, respectively, l_2, l_3 are the distances from the wellhead to the inlet and outlet pipes to the trunk line.

The initial and boundary conditions

$$\left.\frac{\partial P}{\partial t}\right|_{t=0}=-c^2\frac{G}{f_1}\delta(x_1-l_2)+\frac{c^2G_1}{f_1}\delta(x_1-l_3),\quad 0\le x\le l, \tag{2.42}$$

$$P(x,0)|_{t=0}=P_y(0)-2a_1Q_{20}x_1,\qquad 0\le x\le l, \tag{2.43}$$

$$P|_{x_1=0}=P_y(t),\qquad t>0, \tag{2.44}$$

$$P|_{x_1=l_1} = P_2 = \text{const}, \qquad t > 0. \tag{2.45}$$

Allowing for conditions (2.44) and (2.45), we look for a solution of equation (2.41) in the form:

$$P = P_y(t) - \frac{P_y(t) - P_2}{l_1} x + \sum_{i=1}^{n} \varphi_{1i}(t) \Big(\sin \frac{i\pi x_1}{l_1} \Big), \tag{2.46}$$

where $\varphi_{1i}(t)$ is an unknown function depending on time t and l_1 is the length of the pipeline.

Substituting expression (2.46) in formula (2.41) with regard to the initial conditions (2.42), (2.43), in the case $P_2 = \text{const}$, we get

$$Q_2|_{x_1=0} = Q_{20} e^{-2a_1 t} + \frac{1}{l_1} \int_0^t P_y(\tau) \exp[-2a_1(t-\tau)]\, d\tau$$

$$-\sum_{i=1}^{n} \Big(\frac{i\pi}{l_1}\Big) \int_0^t \varphi_{1i}(\tau) \exp[-2a_1(t-\tau)]\, d\tau - \frac{P_2}{2a_1 l_1}(1 - \exp(-2a_1 t)), \tag{2.47}$$

where

$$\varphi_{1i} = \frac{\varphi_2(0)}{\xi_1 - \xi_2}(\exp(\xi_1 t)(2a_1 + \xi_1) - \exp(\xi_2 t)(2a_1 + \xi_2)) + \dot{\varphi}_2(0) \frac{\exp(\xi_1 t) - \exp(\xi_2 t)}{\xi_1 - \xi_2}$$

$$-\sum_{i=1}^{n} \frac{2}{\pi i} \Big(P_y(t) + \frac{\xi_1(2a_1 + \xi_1) \int_0^t P_y(\tau) \exp(\xi_1(t-\tau)) d\tau}{\xi_1 - \xi_2}$$

$$- \frac{\xi_2(2a_1 + \xi_2) \int_0^t P_y(\tau) \exp(\xi_2(t-\tau)) d\tau}{\xi_1 - \xi_2} \Big) + \sum_{i=1}^{n} \frac{2P_y(0)}{\pi i(\xi_1 - \xi_2)}(\exp(\xi_1 t)(2a_1 + \xi_1)$$

$$- \exp(\xi_2 t)(2a_1 + \xi_2)) + \sum_{i=1}^{n} \frac{2\dot{P}_y(0)}{\pi i(\xi_1 - \xi_2)}(\exp(\xi_1 t) - \exp(\xi_2 t))$$

$$\frac{4a_1 c^2}{l_1} \Big[\frac{G_1}{f_1} \sin\Big(\frac{\pi l_3 i}{l_1}\Big) - \frac{G}{f_1} \sin\Big(\frac{\pi l_2 i}{l_1}\Big) \Big] \frac{\xi_2(\exp(\xi_1 t) - 1) - \xi_1(\exp(\xi_2 t) - 1)}{\xi_1 \xi_2 (\xi_1 - \xi_2)}, \tag{2.48}$$

ξ_1 and ξ_2 are the roots of the equation $s^2 + 2a_1 s + \frac{c^2 \pi^2 i^2}{l_1^2} = 0$.

From the continuity condition at the wellhead, allowing for expressions (2.36) and (2.47), we get

$$f_1 \Big(Q_{20} e^{-2a_1 t} + \frac{1}{l_1} \int_0^t P_y(\tau) \exp[-2a_1(t-\tau)]\, d\tau - \sum_{i=1}^{n} \Big(\frac{i\pi}{l_1}\Big) \int_0^t \varphi_{1i}(\tau) \exp[-2a_1(t-\tau)]\, d\tau$$

$$-\frac{P_2}{2a_1 l_1}(1 - \exp(-2a_1 t)) \Big) = f \Big(\frac{G_0}{f} + \frac{1}{l\rho} \int_0^t P_c(\tau) e^{-2a(t-\tau)} d\tau$$

$$-\frac{1}{l\rho} \int_0^t P_y(\tau) e^{-2a(t-\tau)} d\tau + \frac{P_y(0) - P_c(0)}{l\rho} \Big(\frac{1}{2a} - \frac{1}{2a} e^{-2at} \Big)$$

$$+ \sum_{i=1}^{n} 2 \Big(-\frac{2a_{11}}{3l\rho b_{11}} \Big[\int_0^t P_y(\tau) e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau)) d\tau - \int_0^t P_c(\tau) e^{-a_{11}(t-\tau)} \sin(b_{11}(t-\tau)) d\tau \Big]$$

$$+ \frac{2}{3l\rho} \Big[\int_0^t P_y(\tau) e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau)) d\tau - \int_0^t P_c(\tau) e^{-a_{11}(t-\tau)} \cos(b_{11}(t-\tau)) d\tau \Big]$$

$$+ \frac{2(P_c(0) - P_y(0))}{3l\rho} \Big(\frac{e^{-a_{11} t} \sin(b_{11} t)}{b_{11}} \Big) \Big) \Big). \tag{2.49}$$

Applying the Laplace transform, from the expression (2.49), we get

$$f\left[\frac{G_0}{fs}+\frac{1}{\rho l}\frac{\overline{P}_c}{s+2a}-\frac{1}{\rho l}\frac{\overline{P}_y}{s+2a}-\frac{1}{2al}\frac{(P_y(0)-P_c(0))}{\rho(s+2a)}\right.$$
$$+\frac{1}{2al}\frac{(P_y(0)-P_c(0))}{\rho(s+2a)}-\frac{1}{2al}\frac{(P_y(0)-P_c(0))}{\rho s}$$
$$+\sum_{i=1}^{n}2\Big(\frac{-2a_{11}}{3l\rho b_{11}}\frac{\overline{P}_y}{(s+a_{11})^2+b_{11}^2}+\frac{2}{3l\rho}\frac{\overline{P}_y(s+a_{11})}{(s+a_{11})^2+b_{11}^2}$$
$$-\frac{2}{3l\rho}\frac{\overline{P}_c(s+a_{11})}{(s+a_{11})^2+b_{11}^2}+\frac{2a_{11}}{3l\rho}\frac{\overline{P}_c}{(s+a_{11})^2+b_{11}^2}$$
$$\left.+\frac{2(P_c(0)-P_c(0))}{3l\rho}\frac{1}{(s+a_{11})^2+b_{11}^2}\Big)\right]$$
$$=f_1\left(\frac{Q_{20}}{s+2a_1}+\frac{\overline{P}_y}{l_1(s+2a_1)}-\sum_{i=1}^{n}\left(\frac{i\pi}{l_1}\right)\frac{\overline{\varphi}_{1i}}{(s+2a_1)}-\frac{P_2}{l_1s(s+2a_1)}\right). \tag{2.50}$$

Applying the Laplace transform and taking into account convolution and conversion theorems, from expression (2.50), with regard to (2.40), we get the numerical values of the system parameters:

$$c=c_1=1000m\cdot s^{-1};\ \ \mu=10^{-3}Pa\cdot s;\ \ h=5m;\ \ k=10^{-13}m^2;\ \ \rho=860\ \ kq\cdot m^{-3};$$
$$l=2000m;\ l_1=20000m;\ \ P_c(0)=24\cdot10^6Pa;\ P_0=10^6Pa;\ P_y(0)=2\cdot10^6Pa;\ P_k(0)=27\cdot10^6Pa;$$
$$P_{atm}=10^5Pa;\ \ R_k=100m;\ \ \pi=3,14;\ \ a=10^{-3}s^{-1};\ \ a_1=10^{-3}s^{-1};\ \ m=0.2;$$
$$d=6\cdot10^{-2}m;\ \ d_1=20\cdot10^{-2}m;\ \ r_c=7.5\cdot10^{-2}m;\ \ B(x_\nu)=0.114.$$

For $G_1=0$, $G\neq0$,

$$P_y=2.43926\cdot10^6+24609.16665\exp(-0.000205t)-2.08532\cdot10^6\exp(-0.06458t)$$
$$+2.99813\cdot10^6\exp(-0.2271t)+9.87432\cdot10^5\exp(-0.8846t)$$
$$-2.14414\cdot10^6\exp(-0.1065t)\cos(1.8448t)$$
$$+1.16565\cdot10^6\exp(-0.1065t)\sin(1.8448t)-13727.9\exp(-0.001t)\cos(1.5699t)$$
$$+60799.101\exp(-0.001t)\sin(1.5699t). \tag{2.51}$$

For $G=0$, $G_1\neq0$,

$$P_y=8.87766\cdot10^5+7738.2871\exp(-0.000205t)$$
$$+4.61163\cdot10^5\exp(-0.06479t)+2.28325\cdot10^5\exp(-0.70704t)$$
$$+41225.044\exp(-0.15729t)\cos(1.93743t)+$$
$$+3.74652\cdot10^5\exp(-0.15729t)\sin(1.93743t)$$
$$+4.53997\cdot10^5\exp(-0.00322t)\cos(0.11441t)$$
$$+1.51945\cdot10^6\exp(-0.00322t)\sin(0.11441t). \tag{2.52}$$

Now, from expression (2.40), with regard to formulas (2.51), (2.52) and the above numerical values of the parameters, we get an expression to determine the change of pressure $P_c(t)$ depending on the time t at the bottom hole.

When $G_1=0$, $G\neq0$,

$$P_c=2.40329\cdot10^7+5.96764\cdot10^9\exp(-0.000205\,t)$$
$$+2.22954\cdot10^6\exp(-0.06458\,t)-3.85275\cdot10^6\exp(-0.2271\ t)$$
$$-5.96761\cdot10^9\exp(-0.000205\,t)-6.93203\cdot10^6\exp(-0.8846\,t)$$
$$+40387.8203\exp(-0.1065\,t)\cos(1.8448\,t)$$
$$-1.22229\cdot10^6\exp(-0.1065\ t)\sin(1.8448\,t)+32133.57\exp(-0.001\,t)\cos(1.5699\,t)$$

$$-12056.345\exp(-0.001\,t)\sin(1.5699\,t) + 1.01194\cdot 10^{7}\exp(-1.0319\,t). \qquad (2.53)$$

When $G = 0$, $G_1 \neq 0$,

$$\begin{aligned} P_c &= 2.28836\cdot 10^{7} + 4.67149\cdot 10^{5}\exp(-0.006479\,t) \\ &+5.99364\cdot 10^{6}\exp(-0.70704\,t) - 22957.82302\exp(-0.00020507\,t) \\ &-5.41526\cdot 10^{6}\exp(-0.68715\,t) + 39515.63194\exp(-0.00020516\,t) \\ &+1.56454\cdot 10^{6}\exp(-0.00323\,t)\sin(0.114408\,t) \\ &+1.95962\cdot 10^{5}\exp(-0.00323\,t)\cos(0.114408\,t) \\ &-1.20315\cdot 10^{5}\exp(-0.15729\,t)\cos(1.93743\,t) \\ &+47574.81815\exp(-0.15729\,t)\cos(1.93743\,t). \end{aligned} \qquad (2.54)$$

Numerical calculations were carried out by formulas (2.51), (2.52) and (2.53), (2.54).

The results of numerical calculations were represented in Figures 2, 3, 4 and 5. As can be seen from Figure 2, when connected to the trunk pipeline, at the initial stage of the wellhead pressure pulsates with large amplitude, and then after certain time it damps and stabilizes. The same picture is observed for pressure at the bottom hole with the difference that pressure stabilizes more rapidly. These changes are reflected on the well production rate.

The results of calculation on determining the wellhead $P_y(t)$ and bottom hole pressure $P_c(t)$ in the case of fluid withdrawal from the trunk pipe are represented in Figures 4 and 5. As can be seen in Figures 4 and 5, when withdrawing fluid from the transport line, wellhead and bottom hole pressure also pulsate around the stationary state with damping amplitude. First, they become greater than stationary pressure, then after certain time, they damp and stabilize. This takes out the wells from the stationary state and negatively affects its productivity.

3. Conclusion

A hydrodynamic model of the process of non-stationary motion of fluid in a reservoir-pipeline system under connections and withdrawal of fluid from the acting transport line was structured. The reservoir per unit time were obtained. Numerical calculations were carried out for practical values of the system parameters. Analytical expressions to determine the dynamics of pressures at the wellhead and the bottom hole and also the fluid flow

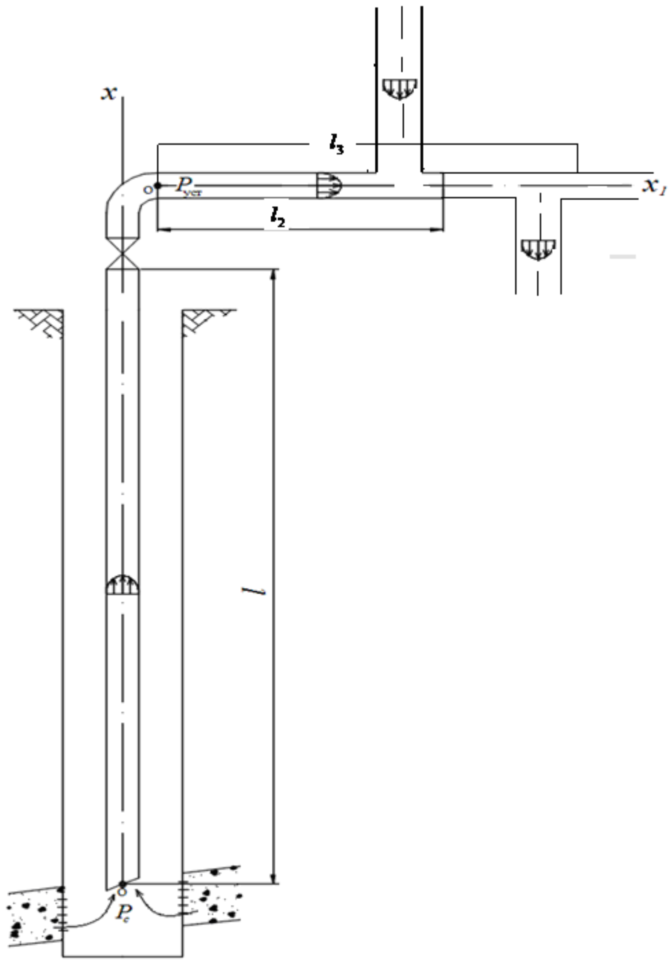

FIGURE 1.

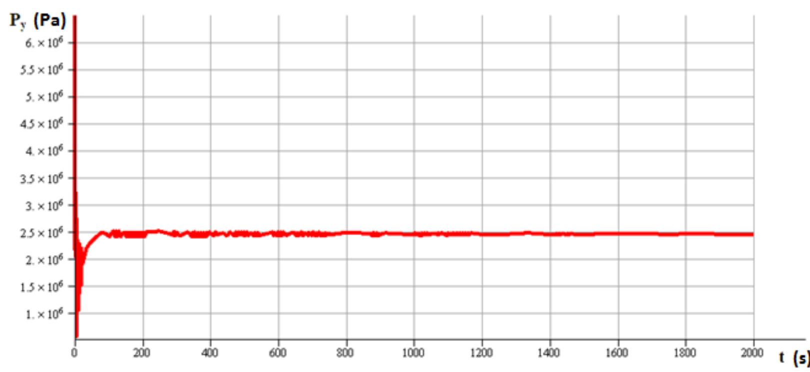

FIGURE 2.

Denotation

P is pressure at any point of the reservoir, Pa; P_k is pressure on reservoir's contour, Pa; P_c is pressure at the bottom hole , Pa; φ_i, φ_{1i} are unknown time-dependent functions; f is the area of the

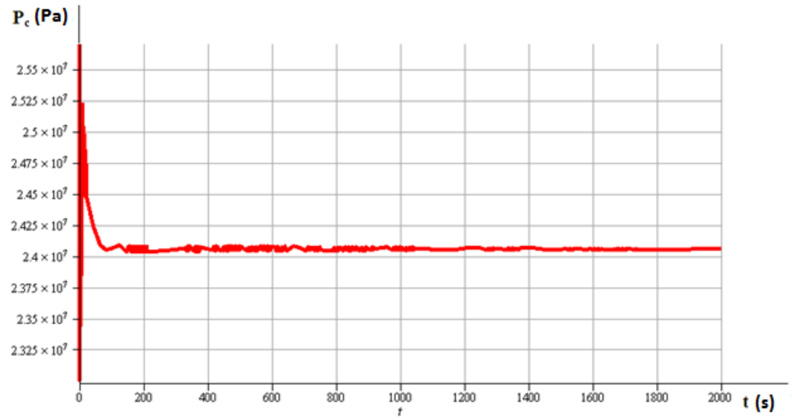

FIGURE 3.

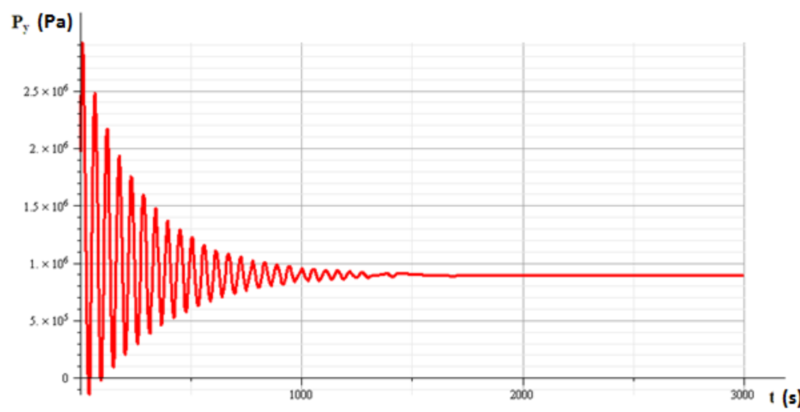

FIGURE 4.

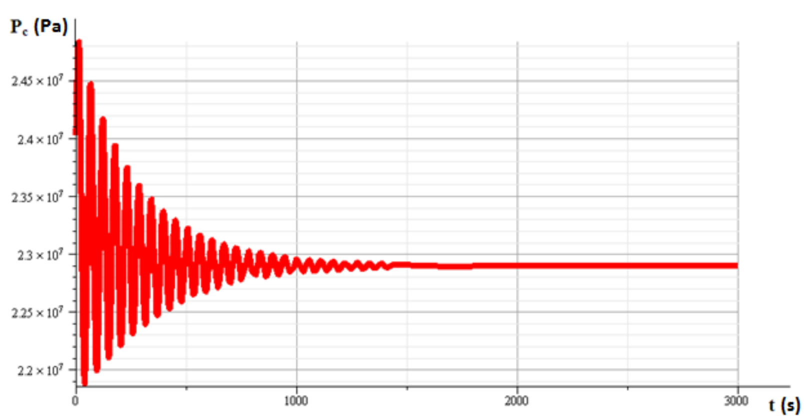

FIGURE 5.

cross-section of the lifting column, m^2; f_1 is the area of the cross-section of the transport pipeline , m^2; h is reservoir's power , m; a, a_1 is drag coefficient, s^{-1}; P_0 is initial pressure in the well, Pa; P_{c_1} is the final value of the pressure at the bottom hole after its change $P_c(0)$, Pa, $\frac{r}{R_k}$ is non-dimensional quantity, R_k is the radius of the reservoir, m; r is a coordinate, m; l is the depth of descent of lifting pipes, m; l_1 is the length of the transport pipeline, m; r_c is the radius of well, m; τ, t is time , A; χ is piezoconductivity coefficient, m^2/s; μ is dynamic viscosity coefficient of fluid, $mPa \cdot s$; β^* is compressibility coefficient, $MP0^{-1}$; k is effective permeability of the reservoir, m^2, J_0 and J_1, Y_0 and Y_1 are the first and second kind Bessel functions of zero and the first order; ρ is fluid density, kq/m^3; U_ν,x_ν are denotation, $\nu = 1, 2, 3, \dots, n$ are natural numbers; ω_i is vibration frequency, s^{-1}; x, x_1 are coordinates; $B(x_v)$, $b(x_v)$, $A_1, \dots, A_5$, s, are denotation; indices: $*$ is upper index, 0 is lower index, k is a contour; A is a well; 0 and 1 are zero and the first orders.

Figure inscriptions

Figure 1 – Calculation scheme.
Figure 2 – Dynamics of change of pressure $P_y(t)$ at the wellhead ($G_1 = 0$, $G \neq 0$).
Figure 3 – Dynamics of change of pressure $P_c(t)$ at the bottom hole ($G_1 = 0$, $G \neq 0$).
Figure 4 – Dynamics of change of pressure $P_y(t)$ at the wellhead ($G = 0$, $G_1 \neq 0$).
Figure 5 – Dynamics of change of pressure $P_c(t)$ at the bottom hole ($G = 0$, $G_1 \neq 0$).

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Azerbaijan, Az 1012, Baku, 88 Zardabi avenue, Scientific-Research Design Institute of “Oil and Gas”
E-mail address: n.agayeva1975@gmail.com