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NEW RESULTS ON SEMI-I-CONVERGENCE

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Abstract. In this article, we use the notions of semi-open, semi-I-open sets and S-I-convergence to show and study other properties on semi-I-convergence. Besides, some basic properties of semi-I-Fréchet–Urysohn space are shown. Moreover, the notions related to semi-I-sequential and semi-I-sequentially open spaces are proved. Furthermore, we show some relations of semi-I-irresolute functions between preserving semi-I-convergence functions and semi-I-covering functions.

1. Introduction

The notion of ideal was introduced by Kuratowski in 1933 [5], an ideal I on a space X is a collection of elements of X which satisfies: (1) $\emptyset \in I$, (2) if $A, B \in I$, then $A \cup B \in I$, and (3) if $B \subset I$ and $A \subset B$, then $A \in I$. This notion has been grown in several concepts of general topology. In 2019, Zhou and Lin [8] used the notion of ideal on the set $\mathbb{N}$ to extend the notion of I-convergence, the results were useful for the developing of this paper. Recently, in 2020, Guevara et.al. [3] have shown some basic properties of S-I-convergent sequences and studied the notions related to the compactness and cluster points by using semi-open sets, furthermore, they have proved that S-I-convergence implies I-convergence for any ideal I on $\mathbb{N}$. On the other hand, in 1963, Levine [6] introduced the concept of semi-open sets in topological spaces, and then in 2005, Hatir and Noiri [4] presented the idea of semi-I-open sets and semi-I-continuous functions in the ideal topological spaces. In this article, we took in the whole the notions mentioned above, define other properties on semi-I-convergence and study the relation between semi-I-sequentially open and semi-I-sequential spaces. Moreover, we define and study some basic properties of preserving semi-I-convergence functions and semi-I-covering functions, furthermore, we prove some relations with semi-I-irresolute functions. Besides, the idea of semi-I-Fréchet–Urysohn spaces is defined.

Throughout this paper, the terms (X, τ) and (Y, σ) mean topological spaces on which no separation axioms are assumed unless otherwise mentioned. Additionally, we sometimes write X or Y instead of (X, τ) or (Y, σ), respectively.

2. Semi-I-convergence

We first introduce some definitions.

Definition 2.1. Let (X, τ) be a topological space, $A \subset X$ and $x \in X$. Then A is said to be semi-neighbourhood if and only if there exits a semi-open set B such that $x \in B \subset A$.

Definition 2.2. A sequence $(x_n : n \in \mathbb{N})$ in a topological space X is called semi-I-convergent to a point $x \in X$, provided for any semi-neighbourhood U of x, it has $A_V = \{n \in \mathbb{N} : x_n \notin V\} \in I$, which is denoted by $s\text{-}I\text{-}\lim\limits_{n\to\infty} x_n = x$ or $x_n \to^{sI} x$, and the point x is called the s-I-limit of the sequence $(x_n : n \in \mathbb{N})$.

Definition 2.3. Let (X, τ) be a topological space and $A \subset X$. Then A is called semi-I-sequentially open if and only if no sequence in $X - A$ has a semi-I-limit in A. That is, the sequence cannot be semi-I-convergent outside of a semi-I-sequentially closed set.

Definition 2.4. Let I be an ideal on $\mathbb{N}$ and X be a topological space, then:

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(1) A subset J of X is said to be semi-I-closed if for each sequence $(x_n : n \in \mathbb{N}) \subseteq J$ with $x_n \to^{sI} x \in X$, then $x \in J$.
(2) A subset V of X is said to be semi-I-open if $X - V$ is semi-I-closed.
(3) X is said to be a semi-I-sequential space if each semi-I-closed set in X is closed.

Definition 2.5. Let (X, τ) be a topological space. Then X is semi-I-sequential, when any set A is semi-open, if and only if it is semi-I-sequentially open.

Now, we show some results.

Lemma 2.1 ([8]). *Let I be an ideal on $\mathbb{N}$ and X be a topological space. If a sequence $(X_n : x \in \mathbb{N})$ I-converges to a point $x \in X$ and $(y_n : n \in \mathbb{N})$ is a sequence in X with $\{n \in \mathbb{N} : x_n \neq y_n\} \in I$, then the sequence $(y_n : n \in \mathbb{N})$ I-converges to $x \in X$*

Lemma 2.2 ([8]). *Let $I \subseteq J$ be two ideals of $\mathbb{N}$. If $(x_n : n \in \mathbb{N})$ is a sequence in a topological space X such that $x_n \to^I x$, then $x_n \to^J x$.*

Lemma 2.3. *Let (X, τ) be a topological space. Then $B \subset X$ is semi-I-sequentially open if and only if every sequence with semi-I-limit in B has all, but finitely many, terms in B, where the index set of the part in B of the sequence does not belong to I.*

Proof. Suppose that B is not a semi-I-sequentially open, then there is a sequence with terms in $X - B$, but semi-I-limit in B. Conversely, suppose that $(x_n : n \in \mathbb{N})$ is a sequence with infinitely many terms in $X - B$ such that semi-I-converges to $y \in A$ and the index set of the part in B of the sequence does not belong to I. Then $(x_n : n \in \mathbb{N})$ has a subsequence in $X - B$ that has still to converge to $y \in B$ and so, B is not semi-I-sequentially open. □

Lemma 2.4. *Let I and J be two ideals of $\mathbb{N}$, where $I \subseteq J$ and X is a topological space. If $V \subseteq X$ is semi-J-open, then it is semi-I-open.*

Proof. Let $V \subseteq X$ be semi-J-open. Then $X - V$ is semi-J-open, if $(x_n : n \in \mathbb{N})$ is a sequence in $X - V$ with $x_n \to^{sI} x$, thus by Lemma 2.2, it has $x \in X - V$. Therefore, V is semi-I-open. □

Corollary 2.1. *Let I and J be two ideals of $\mathbb{N}$. If a topological space X is semi-I-sequential, then it is semi-J-sequential.*

Lemma 2.5. *Let I be an ideal on $\mathbb{N}$ and X be a topological space. If a sequence $(X_n : x \in \mathbb{N})$ is semi-I-convergent to a point $x \in X$ and $(y_n : n \in \mathbb{N})$ is a sequence in X with $\{n \in \mathbb{N} : x_n \neq y_n\} \in I$, then the sequence $(y_n : n \in \mathbb{N})$ is semi-I-convergent to $x \in X$.*

Proof. The proof is followed by Lemma 2.1 and Definition 2.2. □

Lemma 2.6. *Let X be a topological space X, $A \subset X$ and I be an ideal on $\mathbb{N}$. Then the following statements are equivalent.*

(1) *A is semi-I-open.*
(2) *$\{n \in \mathbb{N} : x_n \in A\} \notin I$ for each sequence $(x_n : n \in \mathbb{N})$ in X with $x_n \to^{sI} x \in A$.*
(3) *$|\{n \in \mathbb{N} : x_n \in A\}| = \theta$ for each sequence $(x_n : n \in \mathbb{N})$ in X with $x_n \to^{sI} x \in A$.*

Proof. (1) $\Rightarrow$ (2) : Suppose that A is a semi-I-open set of X and let $(x_n : n \in \mathbb{N})$ be a sequence in X satisfying $x_n \to^{sI} x \in A$. Now, take $N_0 = \{n \in \mathbb{N} : x_n \in A\}$. If $N_0 \in I$, then $N_0 \neq \mathbb{N}$ and so, $A \neq X$. Now, take a point $a \in X - A$ and define the sequence $(y_n : n \in \mathbb{N})$ in X by $y_n = a, n \in N_0$, thus $y_n = x_n$, $n \notin N_0$. By Lemma 2.5, the sequence $(y_n : n \in \mathbb{N})$ semi-I-converges to x. We can see that $X - A$ is semi-I-closed and $(y_n)_{n \in \mathbb{N}} \subseteq X - A$, as a consequence, $x \in X - A$, but this is a contradiction. Therefore, $N_0 \notin I$.

The implication (2) $\Rightarrow$ (3) follows from the notion that the ideal I is admissible.

Now, let us show the following implication. (3) $\Rightarrow$ (1) : Let A be nonsemi-I-open in X. Then $X - A$ is not semi-I-closed and there is a sequence $(x_n : n \in \mathbb{N}) \subseteq X - A$ with $x_n \to^{sI} x \in A$ and this is a contradiction. □

Theorem 2.1. *Every semi-I-sequential space is hereditary with respect to semi-I-open (semi-I-closed) subspaces.*

Proof. Let X be a semi-I-sequential space. Suppose now that A is a semi-I-open set of X. Then A is semi-open in X. Now, we can see that A is semi-I-sequential. Let V be a semi-I-open set in A, thus V is semi-open in X. Indeed, by Definition 2.5, if we show that V is semi-I-open in X, this will be sufficient. Now, suppose that there is a point $y \in Y - V$ and take an arbitrary $x \in V$ and a sequence $(x_n : n \in \mathbb{N}) \subseteq X$ with $x_n \to^{sI} x$ in X. Since A is semi-open in X and $x \in A$, the set $\{n \in \mathbb{N} : x_n \notin A\} \in I$. We define the sequence $y_n : n \in \mathbb{N})$ in X by $y_n = x_n$, $x_n \in A$, $y_n = y$, $x_n \notin A$. By Lemma 2.5, the sequence $(y_n : n \in \mathbb{N})$ is semi-I-convergent to x. Since $|\{n \in \mathbb{N} : x_n \notin V\}| = |\{n \in \mathbb{N} : y_n \notin V\}|$ and by Lemma 2.6, V is semi-I-open in X.

Now, let A be a semi-I-closed set of X. Then A is semi-closed in X. For any semi-I-closed set J of A we have to show that J is semi-closed in X, but since X is a semi-I-sequential space, it suffices for J to be semi-I-closed in X. Hence, let $(x_n : n \in \mathbb{N})$ be an arbitrary sequence in J with $x_n \to^{sI} x \in X$. Thus we obtain that $x \in J$. Indeed, since A is semi-closed, therefore $x \in A$ and so, $x \in J$, since J is a semi-I-closed set of A. □

Theorem 2.2. *Semi-I-sequential spaces are preserved by the topological sums.*

Proof. Let $\{X_\delta\}_{\delta\in\Delta}$ be a family of semi-I-sequential spaces. Take $X = \bigoplus_{\delta\in\Delta} X_\delta$, being the topological sum of $\{X_\delta\}_{\delta\in\Delta}$. We now show that the topological sum is a semi-I-sequential space. Let J be a semi-I-closed set in X. For each $\delta \in \Delta$, since X_δ is semi-closed in X, $J \cap X_\delta$ is semi-I-closed in X. We can see that $J \cap X_\delta \subseteq X_\delta$ and $J \cap X_\delta$ is semi-I-closed in X_δ. By the assumption, we have that $J \cap X_\delta$ is semi-closed in X_δ. By the definition of topological sums, we get that J is semi-closed in X. Therefore, the topological sum X is a semi-I-sequential space. □

Remark 2.1. The union of a family of semi-I-open sets is a topological space which is semi-I-open. Therefore, the intersection of finitely many sequentially semi-I-open sets is sequentially semi-I-open

Definition 2.6. Let I be an ideal on $\mathbb{N}$ and A be a subset of a topological space X. A sequence $(x_n : n \in \mathbb{N})$ in X is I-eventually in A [8] if there is $B \in I$ such that for all $n \in \mathbb{N} - B$, $x_n \in A$.

Proposition 2.1. *Let I be a maximal ideal on $\mathbb{N}$ and X be a topological space. Then A is a subset of X, where A is semi-I-open if and only if each semi-I-convergent sequence in X, converging to a point of A is I-eventually in A.*

Proof. Let A be a semi-I-open and $x_n \to^{sI} x \in A$. Since I is maximal, by Lemma 2.6, $B = \{n \in \mathbb{N} : x_n \notin A\} \in I$. Therefore, for each $n \in \mathbb{N} - B$, $x_n \in A$, i.e., the sequence $(x_n : n \in \mathbb{N})$ is I-eventually in A. □

Theorem 2.3. *Let I be a maximal ideal of $\mathbb{N}$ and X be a topological space. If V, W are two semi-I-open sets of X, then $V \cap W$ is semi-I-open.*

Proof. It will be shown that every semi-I-convergent sequence converging to a point in $V \cap W$ is I-eventually in it. Let $(x_n : n \in \mathbb{N})$ be a sequence in X such that $x_n \to^{sI} x \in V \cap W$. There are $A, S \in I$ such that for each $n \in \mathbb{N} - A$, $x_n \in V$ and for each $n \in \mathbb{N} - S$, $x_n \in W$. Since $A \cup S \in I$ and for each $n \in \mathbb{N} - (A \cup S)$, $x_n \in V \cap W$, we have $V \cap W$ is a semi-I-open set. □

3. Further Properties

3.1. **Semi-I-irresolute functions.** In this part, we introduce semi-I-irresolute functions and show some relations between continuous and semi-I-continuous functions.

Definition 3.1 ([1]). Let $f : (X, \tau) \to (Y, \sigma)$ be the functions. f is called sequentially continuous, provided V is sequentially open in Y, then $f^{-1}(V)$ is sequentially open in X.

Definition 3.2. Let I be an ideal on $\mathbb{N}$, $(X, \tau), (Y, \sigma)$ be topological spaces and $f : (X, \tau) \to (Y, \sigma)$ be a function, then:

(1) f is said to be preserving semi-I-convergence, provided for each sequences $(x_n : n \in \mathbb{N})$ in X with $x_n \to^{sI} x$, the sequence $(f(x_n) : n \in \mathbb{N})$ is semi-I-convergent to $f(x)$.
(2) [4] f is said to be semi-I-irresolute if for each semi-I-open V in Y, then $f^{-1}(V)$ is semi-I-open in X.

Lemma 3.1 ([4]). *Every semi-I-irresolute function is semi-I-continuous.*

Theorem 3.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f is continuous, then f preserves semi-I-convergence.*

Proof. Suppose that f is continuous and let $(x_n : n \in \mathbb{N})$ be a sequence in X such that $x_n \to^{sI} x \in X$. Now, let V be an arbitrary semi-neighbourhood of $f(x)$ in Y. Since f is continuous, $f^{-1}(V)$ is a semi-neighbourhood of x. Therefore, we have $\{n \in \mathbb{N} : x_n \notin f^{-1}(V)\} \in I$. We can see that $\{n \in \mathbb{N} : f(x_n) \notin V\} = \{n \in \mathbb{N} : x_n \notin f^{-1}(V)\}$. This implies that $\{n \in \mathbb{N} : f(x_n) \notin V\} \in I$. Hence, $f(x_n) \to^{sI} f(x)$. □

Theorem 3.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f preserves the semi-I-convergence, then f is semi-I-irresolute.*

Proof. Suppose that f preserves semi-I-convergence and J is an arbitrary semi-I-closed set in Y. Let $(x_n : n \in \mathbb{N})$ be a sequence in $f^{-1}(J)$ such that $x_n \to^{sI} x \in X$. By the assumption, we have $f(x_n) \to^{sI} f(x)$. Since $(f(x_n) : n \in \mathbb{N}) \subseteq J$ and J is semi-I-closed in Y, hence $f(x) \in J$, i.e., $x \in f^{-1}(J)$. Therefore, $f^{-1}(J)$ is semi-I-closed in X and then f is semi-I-irresolute. □

Proposition 3.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f preserves the semi-I-convergence, then f is semi-I-continuous.*

Proof. The proof is followed by Lemma 3.1 and Theorem 3.2. □

Theorem 3.3. *Let I be a maximal ideal on $\mathbb{N}$. Then a function $f : (X, \tau) \to (Y, \sigma)$ is semi-I-irresolute if and only if it preserves semi-I-convergent sequences.*

Proof. Assume that f is semi-I-irresolute and a sequence $x_n \to^{sI} x$ in X. We have to show that $f(x_n) \to^{sI} f(x)$ in Y. Now, let V be a semi-neighbourhood of $f(x)$. Then $x \in f^{-1}(V)$ is semi-I-open in X, because V is semi-I-open in Y. Hence, there is $B \in I$ such that for all $n \in \mathbb{N} - B$, $x_n \in f^{-1}(V)$. This means that for all $n \in \mathbb{N} - B$, $f(x_n) \in V$. Therefore, $\{n \in \mathbb{N} : f(x_n) \notin V\} \in I$ and hence, $f(x_n) \to^{sI} f(x)$. □

Theorem 3.4. *Let X be a semi-I-sequential space and $f(X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent.*

(1) *f is continuous.*
(2) *f preserves semi-I-convergence.*
(3) *f is semi-I-irresolute.*

Proof. (1) ⇔ (2) was proved in Theorems 3.1 and 3.2.

(3) ⇒ (1) : Let f be semi-I-irresolute and J be an arbitrary semi-closed set in Y. Then J is semi-I-closed in Y. Since f is semi-I-irresolute, $f^{-1}(J)$ is semi-U-closed in X. By the assumption, we find that $f^{-1}(J)$ is semi-closed in X. Therefore, f is continuous. □

Proposition 3.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and X be a semi-I-sequential space. Then the following statements are equivalent.*

(1) *f is continuous.*
(2) *f is semi-I-continuous.*

Proof. The proof is followed by Proposition 3.1 and Theorem 3.4. □

Lemma 3.2. *Let X be a semi-I-sequential space, then the function $f : (X, \tau) \to (Y, \sigma)$ is continuous if and only if it is sequentially continuous.*

Proof. Let X be a semi-I-sequential space, then every semi-I-closed set is closed, by [1] who proved that f is continuous if and only if f is sequentially continuous, indeed we have completed the proof. □

Corollary 3.1. *Let X be a semi-I-sequential space and for a function $f : (X, \tau) \to (Y, \sigma)$ the following statements are equivalent.*

(1) *f is continuous.*
(2) *f preserves semi-I-convergence.*
(3) *f is semi-I-continuous.*
(4) *f is sequentially continuous.*

Proof. (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3) was proved in Theorem 3.4, by Lemma 3.2, we have (1) $\Leftrightarrow$ (4). ☐

Lemma 3.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and X be a semi-I-sequential space. Then the following statements are equivalent.*

(1) *f is sequentially continuous.*
(2) *f is semi-I-continuous.*

Proof. The proof is followed by Proposition 3.2 and Corollary 3.1. ☐

3.2. **Semi-I-irresolute and semi-I-covering functions.** Continuity and sequentially continuity are the ones of the most important tools for studying sequential spaces [7]. In this part, we define the concept of semi-I-covering functions and show some of their properties.

Definition 3.3 ([1])**.** Let $f : (X, \tau) \to (Y, \sigma)$ be a topological space. Then f is said to be sequentially continuous, provided $f^{-1}(V)$ is sequentially open in X, then V is sequentially open in Y.

Definition 3.4 ([1])**.** Let $f : (X, \tau) \to (Y, \sigma)$ be a topological space. Then f is said to be sequence-covering if, whenever $(y_n : n \in \mathbb{N})$ is a sequence in Y covering to y in Y, there exits a sequence $(x_n : n \in \mathbb{N})$ of points $x_n \in f^{-1}(y_n)$ for all $n \in \mathbb{N}$ and $x \in f^{-1}(y)$ such that $x_n \to x$.

Definition 3.5. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is said to be semi-I-covering if, whenever $(y_n : n \in \mathbb{N})$ is a sequence in Y, semi-I-converging to y in Y, there exits a sequence $(x_n : n \in \mathbb{N})$ of points $x_n \in f^{-1}(y_n)$ for all $n \in \mathbb{N}$ and $x \in f^{-1}(y)$ such that $x_n \to^{sI} x$.

Theorem 3.5. *Every semi-I-covering function is semi-I-irresolute.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a function and f be a semi-I-covering function. Assume now that V is a non-semi-I-closed in Y. Then there exits a sequence $(y_n : n \in \mathbb{N}) \subseteq V$ such that $y_n \to^{sI} y \notin V$. Since f is semi-I-covering, there exits a sequence $(x_n : n \in \mathbb{N})$ of points $x_n \in f^{-1}(y_n)$ for all $n \in \mathbb{N}$ and $x \in f^{-1}(y)$ such that $x_n \to^{sI} x$. We can see now that $(x_n : n \in \mathbb{N}) \subseteq f^{-1}(V)$ and so, $x \notin f^{-1}(V)$, therefore $f^{-1}(V)$ is non-semi-I-closed. As a conclusion, f is semi-I-irresolute. ☐

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements hold.*

(1) *If X is a semi-I-sequential space and f is continuous, then Y is a semi-I-sequential space and semi-I-irresolute.*
(2) *If Y is a semi-Y-sequential space and f is semi-I-irresolute, then f is continuous.*

Proof. (1) Let X be a semi-I-sequential space and f be continuous. Suppose that V is semi-I -open in Y. Since f is a continuous function and X is a semi-I-sequential space, take an arbitrary sequence $(x_n : n \in \mathbb{N}) \subseteq X$ such that $x_n \to^{sI} x \in f^{-1}(V)$ in X. Since f is a continuous function, by Theorem 3.1, $f(x_n) \to^{sI} f(x) \in V$. Now, since V is semi-I-open in Y and by Lemma 2.6, we have $|\{n \in \mathbb{N} : f(x_n) \in V\}| = \theta$, i.e., $|\{n \in \mathbb{N} : x_n \in f^{-1}(V)\}| = \theta$, therefore, $f^{-1}(V)$ is semi-I-open in X.

Assume now that $V \subseteq Y$ such that $f^{-1}(V)$ is semi-I-open in X. Then $f^{-1}(V)$ is an open set of X, since X is semi-I-sequential space. As is well know, f is continuous, then V is open in Y. Hence, f is continuous.

(2) Let Y be a semi-I-sequential space and f be semi-I-irresolute. If $f^{-1}(V)$ is an open set of X, then $f^{-1}(V)$ is a semi-I-open set of X. Since f is semi-I-irresolute, V is a semi-I-open set of Y. Now, we know that Y is a semi-I-sequential space and so, V is an open set of Y. Therefore, f is continuous. ☐

By Theorems 3.4 and 3.6, we have the following result.

Corollary 3.2. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function, then f is continuous if and only if f is semi-I-irresolute and Y is a semi-I-sequential space.*

3.3. **Semi-I-Fréchet–Urysohn spaces.** A topological space X is said to be Fréchet–Urysohn [2] if for each $A \subseteq X$ and each $x \in Cl(A)$, there is a sequence in A converging to the point x in X. Now, in this part, we introduce the notion of semi-I-Fréchet–Urysohn and show a short result.

Definition 3.6. Let (X,τ) be a topological space. Then X is said to be semi-I-Fréchet–Urysohn or, simply, S-I-FU, if for each $A \subseteq X$ and each $x \in sCl(A)$, there exits a sequence in A, semi-I-converging to the point $x \in X$.

Lemma 3.4. *For two ideals I and J on $\mathbb{N}$, where $I \subseteq J$, if X is a S-I-FU-space, then it is a semi-J-FU-space.*

Proof. Let A be a subset of X and $x \in sCl(A)$. Since X is a S-I-FU-space, then there exits a sequence $(x_n : n \in \mathbb{N})$ in A such that $x_n \to^{sI} x$. As a consequence, $x_n \to^{sI} x$ in X, and so, X is a semi-J-FU-space. □

Theorem 3.7. *Let (X,τ) be a topological space. If X is a S-I-FU-space, then X is a semi-I-sequential space.*

Proof. Let $\{A_\delta : \delta \in \Delta\}$ be a family of semi-I-closed subsets of X, where $\delta \in \Delta \in X$, since X is a S-I-FU-space, by Definition 3.6, $A_\delta \subseteq X$ and each $x \in sCl(A_\delta)$. Now, since A_δ is semi-I-closed, $sCl(A_\delta) = A_\delta \in Cl(A)$, but by Definition 3.6, there exits a sequence semi-I-converging to the point $x \in sCl(A) \in Cl(A) \in X$, therefore $\{A_\delta : \delta \in \Delta\}$ is a closed set of X. As a consequence, X is a semi-I-sequential space. □

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