

Frame completions of conditional frames

(report on a recent communication by B. Banaschewski)

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A year ago at the Nashville TACL I was talking about our work with Guram Bezhanishvili and David Gabelaia concerning an elucidation of the Funayama embedding of certain lattices (L) into complete Boolean algebras (B) using topological (Priestley and Esakia) dualities. One thing we could not achieve by that time was a universal characterization of the embedding $L \rightarrow B$.

Recently Bernhard Banaschewski, who attended my talk then, has sent to Guram an illuminating note which in particular makes completely obvious what this universal property is. The “only” thing that was missing was to find the correct category in which the embedding must take place. With his usual virtuoso elegance Banaschewski has found the only perfectly fitting, very clear and natural context.

Banaschewski considers what he calls conditional frames – lattices in which finite meets distribute over all existing joins. He then proposes what is the most natural notion of morphism between conditional frames – join-preserving lattice homomorphisms (i. e. those preserving all existing joins). He then constructs in the resulting category a universal embedding of any conditional frame into a frame (which is obviously nothing else but a complete conditional frame). Finally this enables him to characterize the Funayama embedding of a conditional frame $L \rightarrow B$ as a join-preserving embedding into a complete Boolean algebra generated by the image of L .

I will provide some details of the Banaschewski construction and tell a couple of words about its dual topological meaning.