

On algebraic analysis of temporal Heyting calculus

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Modalized Heyting calculus **mHC**, introduced by Leo Esakia in [1], is the augmentation of the intuitionistic logic **Int** by a modal operator \Box . This modalized Heyting calculus is a weakening of the proof-intuitionistic logic **KM** of Kuznetsov and Muravitsky by discarding Löb's axiom. There is an exact embedding of **mHC** into the modal system **K4.Grz**. Temporal Heyting calculus **tHC** is a temporal enrichment of **mHC**. This calculus was also introduced by Leo Esakia in [1]. The temporal Heyting calculus **tHC** is defined on the basis of **mHC** with additional axioms for the "adjoint" modality \Diamond , namely:

$$\text{t1) } p \rightarrow \Box \Diamond p$$

$$\text{t2) } \Diamond \Box p \rightarrow p$$

$$\text{t3) } \Diamond(p \vee q) \rightarrow (\Diamond p \vee \Diamond q)$$

$$\text{t4) } \Diamond \perp \rightarrow \perp$$

and an additional rule:

$$\frac{p \rightarrow q}{\Diamond p \rightarrow \Diamond q}$$

Algebraic models of **mHC** are **fHA**-algebras (frontal Heyting algebras). In [2] Castiglioni, Sagastume and San Martin have extended Heyting duality to the category **fHA**.

We investigate the variety of temporal Heyting algebras **tHA**, which represent algebraic models of temporal Heyting calculus **tHC**. We have the following results:

- We develop a theory of temporal Heyting algebras.
- We generalize Heyting duality to the category **tHA**.
- We characterize subdirectly irreducible and simple **tHA**-algebras.

REFERENCES

- [1] L. Esakia, *The modalized Heyting calculus: a conservative modal extension of the intuitionistic logic*. Journal of Applied Non-Classical Logics **16** (2006), 349–346.
- [2] Jose L. Castiglioni, Marta S. Sagastume, Hernan J. San Martin, *On frontal Heyting algebras*. Reports on Mathematical Logic **45** (2010), 201–224.