Camp models for Moore spaces and duality for Stone bases

Marcel Erné

Faculty for Mathematics and Physics
Leibniz University Hannover
D-30167 Hannover, Germany
e-mail: erne@math.uni-hannover.de

Abstract

A *camp* is a **c**onditionally up-complete, **a**lgebraic, **m**aximized and **p**oint-generated poset, the last condition meaning that every compact element is a meet of maximal elements. Every T_1-base space (that is, every T_1-space with a distinguished base) is represented as the maximal point space of an essentially unique camp. Moreover, there is a duality between the category of camps with suitable morphisms and the category of T_1-base spaces.

Of particular interest for domain theory and its applications, both in topology and in computer science, are those camps which are directed-complete (damps for short). A T_1-space has such a damp model iff it has a base not containing any free filter base (with empty intersection).

Many interesting spaces occurring in classical topology even possess so-called complete bases, that is, bases that contain no non-free filter bases except neighborhood bases. For example, all complete Aronszajn spaces, a fortiori all complete Moore spaces and, in particular, all completely metrizable spaces have such complete bases and consequently damp models, which admit a precise order-theoretical description not referring to any topological notions. More generally, every (not necessarily complete) metric space has a camp model in which all principal ideals are countable and non-maximal principal ideals are even finite.

Among all T_1-spaces it is exactly the zero-dimensional ones that occur as so-called umbrellas in camps; topologically, such umbrellas are subspaces of (perhaps not all) maximal points on which the Scott topology induces the same relative topology as the Lawson topology. More than that: every Stone space is represented as the subspace of *all* maximal points in a damp with the property stated before. The latter representation gives rise to an equivalence and a duality for so-called Stone bases, extending the classical Stone duality. This includes a purely order-theoretical description of clopen bases for Stone spaces and, algebraically, of finitary meet bases of Boolean lattices in terms of maximal ideals. Only that very last step requires choice principles; in particular, we have a choice-free Stone duality that becomes the classical one if and only if the Boolean Prime Ideal Theorem holds.