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APPLICATION OF AN EXTENSION OF THE LEBESGUE INTEGRAL TO THE INVESTIGATION OF A BOUNDARY VALUE PROBLEM OF LINEAR CONJUGATION

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Abstract. Some properties of Cauchy type $\widetilde{L}$-integrals on Carleson simple closed curves are derived. $\widetilde{L}$-integral is an extension of Lebesgue integral. The obtained results are applied to the solution of boundary value problems of linear conjugation in the class of Cauchy type integrals.

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1. Let Γ be a simple, closed, rectifiable curve dividing a complex plane into domains $D^+ = \operatorname{Int}\Gamma$ and $D^- = \operatorname{Ext}\Gamma$. Denote by $F^+(t)(F^-(t))$ an angular boundary value at the point $t \in \Gamma$ of an analytic in $D^+(D^-)$ function F. If F is representable by the Cauchy-Lebesgue type integral

$$F(z) = K_\Gamma(f)(z) = \frac{1}{2\pi i}\int_\Gamma \frac{f(t)dt}{t-z}, \quad f \in L(\Gamma), \quad z \notin \Gamma, \tag{1.1}$$

then, as it is known, for almost all $t \in \Gamma$

$$F^\pm(t) = \pm\frac{1}{2}f(t) + \frac{1}{2}(S_\Gamma f)(t), \tag{1.2}$$

where $S_\Gamma(f)(t) = \frac{1}{\pi i}\int_\Gamma \frac{f(\tau)d\tau}{\tau - t}$, $t \in \Gamma$. In particular, from (1.2) it follows that

$$F^+(t) - F^-(t) = f(t). \tag{1.2$'$}$$

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In the present paper we establish some properties of functions which can be represented in the form

$$F(z)=\begin{cases} K_\Gamma(f_1)(z), & z\in D^+,\\ K_\Gamma(f_2)(z), & z\in D^-,\end{cases}\qquad f_i\in L(\Gamma),\quad i=1,2.$$

In particular, we prove that the class of such functions coincides with the class of functions representable in $D^+\cup D^-$ by the Cauchy type $\widetilde{L}$-integral, where $\widetilde{L}$-integral is certain extension of the Lebesgue integral and for the Cauchy type $\widetilde{L}$-integral the analogue of formula (1.2′) is valid. The obtained results are applied to the investigation of one particular case of a boundary value problem of linear conjugation.

We shall say that that Γ satisfies condition (C) if the following statement is valid: If the limiting function F^* formed by angular boundary values of the Cauchy-Lebesgue type integral (1.1) is summable, then $\int_\Gamma F^+(t)dt=0$ (the Cauchy theorem). The latter equality means that if f and $S_\Gamma f$ are summable on Γ, then

$$\int_\Gamma (S_\Gamma f)(t)dt=-\int_\Gamma f(t)dt. \tag{1.3}$$

Condition (C) is satisfied, for example, by Carleson curves, i.e. the curves for which $\sup_{\zeta\in\Gamma} l_\zeta(r)=O(r)$, $l_\zeta(r)$ is a linear measure (length) of the set $\{t\in\Gamma;|\zeta-t|<r\}$ (see [1]), particularly by piecewise smooth curves and also bycurves satisfying the condition: for every pair of points t', $t''\in\Gamma$

$$s(t',t'')\le k|t'-t''|, \tag{1.4}$$

where $s(t',t'')$ is the least arc of the curve Γ with the ends t' and t'', and k is a constant depending only on Γ.

2. Let Γ satisfy the condition (C). We shall say that the function f is $\widetilde{L}$-integrable on Γ (see also [2]) if it can be represented as

$$f(t)=f_1(t)+S_\Gamma(f_2)(t),\quad f_1,\quad f_2\in L(\Gamma). \tag{2.1}$$

A number $\widetilde{L}(f)\equiv(\widetilde{L})\int_\Gamma f(t)dt\equiv\int_\Gamma[f_1(t)-f_2(t)]dt$ will be called a $\widetilde{L}$-integral of f with respect to Γ.

The correctness of the definition of the $\widetilde{L}$-integral (i.e. independence of the value $\widetilde{L}(f)$ from the representation (2.1)) follows from (1.3).

Indeed, let besides (2.1) we have

$$f(t)=g_1(t)+S_\Gamma(g_2)(t),\quad g_1,\quad g_2\in L(\Gamma). \tag{2.2}$$

By the definition, $(\widetilde{L})\int_\Gamma f(t)dt=\int_\Gamma[g_1(t)-g_2(t)]dt$. It follows from (2.1) and (2.2) that $f_1(t)-g_1(t)+S_\Gamma(f_2-g_2)(t)=0$. Then by equality (1.3)

$$\int_\Gamma[f_1(t)-g_1(t)]dt=-\int_\Gamma S_\Gamma(f_2-g_2)(t)dt=\int_\Gamma[f_2(t)-g_2(t)]dt$$

and hence $\int_\Gamma[f_1(t)-f_2(t)]dt=\int_\Gamma[g_1(t)-g_2(t)]dt$.

3. Theorem 1. *Let* Γ *satisfy condition* (C), $f \in L(\Gamma)$, φ *be bounded on* Γ *and*

$$\int_\Gamma \Big|\frac{\varphi(\tau)-\varphi(t)}{\tau - t}\Big||d\tau| < M, \quad t \in \Gamma. \tag{3.1}$$

Then $\varphi S_\Gamma(f) \in \widetilde{L}(\Gamma)$ *and*

$$(\widetilde{L})\int_\Gamma \varphi(t)S_\Gamma(f)(t)dt = -\int_\Gamma f(t)S_\Gamma(\varphi)(t)dt. \tag{3.2}$$

Proof. Consider the equality

$$\varphi(t)S_\Gamma(f)(t)dt = \frac{1}{\pi i}\int_\Gamma f(\tau)\frac{\varphi(t)-\varphi(\tau)}{\tau - t}d\tau + S_\Gamma(\varphi f)(t). \tag{3.3}$$

By condition (3.1), the first summand on the right-hand side of equality (3.3) is the summable on Γ function. Hence $\varphi S_\Gamma(f) \in \widetilde{L}(\Gamma)$. Next, taking the $\widetilde{L}$-integral from the both parts of equality (3.3) and changing the order of integration in the repeated integral, we obtain (3.2). ■

Remark. If Γ is the Carleson curve and $0 < \alpha \le 1$, then the function $I(t) = \int_\Gamma |\tau - t|^{\alpha-1}|d\tau|$ is bounded on Γ and hence the function ψ satisfying on Γ the Hölder condition with the exponent $\alpha - \psi \in H_\Gamma(\alpha)$ satisfies also condition (3.1), (see [3]).

4. Let Γ satisfy condition (C) and $f \in \widetilde{L}(\Gamma)$. Then by Theorem 1 in the domain $D^+ \cup D^-$ we can define the analytic function

$$F(z) = \frac{1}{2\pi i}(\widetilde{L})\int_\Gamma \frac{f(t)dt}{t-z}, \tag{4.1}$$

which will be called the Cauchy type $\widetilde{L}$-integral.

Theorem 2. *The Cauchy type* $\widetilde{L}$*-integral* (4.1) *coincides in the domains* D^+ *and* D^- *with the Cauchy-Lebesgue type integral, the boundary functions* F^+ *and* F^- *are* $\widetilde{L}$*-integrable on* Γ *and for almost all* $t \in \Gamma$

$$F^+(t) - F^-(t) = f(t). \tag{4.2}$$

Proof. Let $f = f_1 + S_\Gamma(f_2)$, where $f_1, f_2 \in L(\Gamma)$. Then for $z \notin \Gamma$, by Theorem 1 and the following easily verifiable equality

$$S_\Gamma\Big(\frac{1}{\tau - z}\Big)(t) = \begin{cases} -(t-z)^{-1}, & z \in D^+, \\ (t-z)^{-1}, & z \in D^-, \end{cases} \tag{4.3}$$

we can write

$$F(z) = \frac{1}{2\pi i}(\widetilde{L})\int_\Gamma \frac{f(t)dt}{t-z} = \frac{1}{2\pi i}\int_\Gamma \frac{f_1(t)dt}{t-z} -$$

$$-\frac{1}{2\pi i}\int_\Gamma f_2(t)S_\Gamma\Big(\frac{1}{\tau - z}\Big)(t)dt = \begin{cases} K_\Gamma(f_1+f_2)(z), & z \in D^+, \\ K_\Gamma(f_1-f_2)(z), & z \in D^-, \end{cases}. \tag{4.4}$$

In particular, from (4.4) and (1.2) follows (4.2). ■

Theorem 3. *If* Γ *satisfies condition* (C) *and* $f \in L(\Gamma)$, *then the boundary function* F^+ (F^-) *of the Cauchy-Lebesgue type integral* (1.1) *is* $\widetilde{L}$*-integrable on* Γ *and*

$$F(z)=\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma \frac{F^+(t)dt}{t-z},\ z\in D^+ \ \Big(F(z)=-\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma \frac{F^-(t)dt}{t-z},\ z\in D^-\Big).$$

Moreover,

$$(\widetilde{L})\int_\Gamma \beta^+(t)F^+(t)dt=0, \tag{4.5}$$

where β^+ *is the boundary function of the analytic in* D^+ *function* β, *satisfying condition* (3.1).

Proof. We shall prove the theorem for the domain D^+. Let $z\notin\Gamma$. Then by equalities (1.2), (3.2) and (4.3) we can write

$$\begin{aligned}&\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma \frac{F^+(t)dt}{t-z}=\frac{1}{4\pi i}\int_\Gamma\frac{f(t)dt}{t-z}+\frac{1}{4\pi i}(\widetilde{L})\int_\Gamma S_\Gamma(f)(t)\frac{dt}{t-z}=\\&=\frac{1}{4\pi i}\int_\Gamma\frac{f(t)dt}{t-z}-\frac{1}{4\pi i}\int_\Gamma f(t)S_\Gamma\Big(\frac{1}{\tau-z}\Big)(t)dt=\begin{cases}K_\Gamma(f)(z), & z\in D^+,\\ 0, & z\in D^-.\end{cases}\end{aligned} \tag{4.6}$$

Equality (4.5) is proved analogously to (4.6), taking into account that $S_\Gamma(\beta^+)=\beta^+$. ■

Theorem 4. *A class of the Cauchy type* $\widetilde{L}$*-integrals* (4.1) *coincides with a class of functions which are representable in the form*

$$F(z)=\begin{cases}F_1(z), & z\in D^+,\\ F_2(z), & z\in D^-,\end{cases} \tag{4.7}$$

where $F_i(z)=K_\Gamma(f_i)(z)$, $f_i\in L(\Gamma)$, $i=1,2$.

Proof. Let F be representable in the form (4.7). Consider on Γ the function $f(t)=F_1^+(t)-F_2^-(t)$, where $F^+(t)=\frac12 f_1(t)+\frac12 S_\Gamma(f_1)(t)$ and $F_2^-(t)=-\frac12 f_2(t)+\frac12 S_\Gamma(f_2)(t)$. It is obvious that $f\in\widetilde{L}(\Gamma)$, and according to the foregoing theorem we can write for $z\notin\Gamma$.

$$\begin{aligned}&\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma\frac{f(t)dt}{t-z}=\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma\frac{F_1^+(t)dt}{t-z}-\\&-\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma\frac{F_2^-(t)dt}{t-z}=\begin{cases}F_1(z), & z\in D^+\\ F_2(z), & z\in D^-.\end{cases}\end{aligned}$$

The inverse assertion follows from Theorem 2. ■

Corollary. *The Cauchy type $\widetilde{L}$-integral* (4.1) *is representable in D^+ (D) by the Cauchy $\widetilde{L}$-integral.*

Indeed, the Cauchy type $\widetilde{L}$-integral can also be representable in D^+ (D^-) by the Cauchy-Lebesgue type integral by Theorem 4 and hence by the Cauchy $\widetilde{L}$-integral by Theorem 3.

Remark. The function F representable in the form (4.7) cannot always be representable in $D^+ \cup D^-$ by the Cauchy-Lebesgue type integral (see Remark to Theorem 5).

Theorem 5. *Let the functions ϕ and F be representable in D^+ (D^-) by the Cauchy-Lebesgue type integrals* (1.1)*, respectively, with densities φ and f corresponding to the conditions of Theorem* 1*. Then the product ϕF is also representable in D^+ (D^-) by the Cauchy-Lebesgue type integrals.*

Proof. Let $z \in D^+$. Then by Theorem 1

$$(\widetilde{L})\int_\Gamma \frac{\varphi(t)S_\Gamma(f)(t)}{t-z}dt = \frac{1}{\pi i}\int_\Gamma f(\tau)d\tau \int_\Gamma \frac{\varphi(t)dt}{(\tau-t)(t-z)} =$$
$$-\int_\Gamma \frac{f(\tau)S_\Gamma(\varphi)(\tau)}{\tau-z}d\tau + \frac{1}{\pi i}\int_\Gamma \frac{\varphi(t)dt}{t-z}\int_\Gamma \frac{f(\tau)d\tau}{\tau-z}.$$

Consequently,

$$\phi(z)F(z) = \frac{1}{4\pi i}\int_\Gamma \frac{f(\tau)S_\Gamma(\varphi)(\tau)}{\tau-z}d\tau +$$
$$+\frac{1}{4\pi i}(\widetilde{L})\int_\Gamma \frac{\varphi(t)S_\Gamma(f)(t)}{t-z}dt,$$

that is ϕF is representable in D^+ as a sum of the Cauchy-Lebesgue and Cauchy type $\widetilde{L}$-integrals. This implies the validity of the theorem, since the Cauchy-type $\widetilde{L}$-integral is representable in D^+ (D^-) by the Cauchy-Lebesgue type integral as well. ■

Corollary 1. *If the functions ϕ and F are representable in $D^+ \cup D^-$ by the Cauchy type integral respectively with densities φ and $f \in \widetilde{L}(\Gamma)$ satisfying the conditions of Theorem* 1*, then the product ϕF is representable in $D^+ \cup D^-$ by the Cauchy type $\widetilde{L}$-integral.*

Indeed, according to the assertion of Theorem 5, ϕF is representable by the Cauchy-Lebesgue type integral both in D^+ and D^-. Then by Theorem 4, ϕF is representable in $D^- \cup D^-$ by the Cauchy type $\widetilde{L}$-integral.

Remark. Note that in Theorem 5 we deal with the representability in D^+ (or in D^-) by the Cauchy type integral, whereas in Corollary 1 asserts the representability in $\widetilde{L}$ by the Cauchy type integral in $D^+ \cup D^-$.

In the assertion of the corollary the $\widetilde{L}$-integral cannot in general be replaced by the Lebesgue integral even for $f \in L(\Gamma)$. Indeed, let Γ be a unit circumference, $\varphi(t) \equiv 1$

on Γ, and $f \in L(\Gamma)$ such that $S_\Gamma(f) \notin L(\Gamma)$. Then the function

$$\Psi(z) = \frac{1}{2\pi i}\int_\Gamma \frac{dt}{t-z}\frac{1}{2\pi i}\int_\Gamma \frac{f(t)dt}{t-z} = \\ = \begin{cases} K_\Gamma(f)(z), & z \in D^+, \\ 0, & z \in D^- \end{cases}$$

(representable in $D^+ \cup D^-$ by the Cauchy type ($\widetilde{L}$-integral) is not representable in $D^+ \cup D^-$ by the Cauchy-Lebesgue type integral. Otherwise, by formulas (1.2') we would have $\Psi^+(t) - \Psi^-(t) = \frac{1}{2}f(t) + \frac{1}{2}S_\Gamma(f)(t) \in L(\Gamma)$, which is impossible, since $f \in L(\Gamma)$ and $S_\Gamma(f) \notin L(\Gamma)$.

Corollary 2. *Let $F = K_\Gamma(f)$, where $f \in L(\Gamma)$ and β^+ is the boundary function of the bounded, analytic in D^+, function β satisfying the condition* (3.1)*. Then*

$$\frac{1}{2\pi i}(\widetilde{L})\int_\Gamma \frac{\beta^+(t)F^+(t)}{t-z}dt = \beta(z)F(z), \quad z \in D^+.$$

5. Let Γ be the Carleson curve, $G \in H_\Gamma(\alpha)$, $G \neq 0$ and $g \in L(\Gamma)$.

Consider the following boundary value problem: find analytic in $D^+ \cup D^-$ function ϕ representable in terms of

$$\phi(z) = \begin{cases} K_\Gamma(\varphi_1)(z) + P(z), & z \in D^+ \\ K_\Gamma(\varphi_2)(z) + P(z), & z \in D^-, \end{cases} \tag{5.1}$$

where $\varphi_i \in L(\Gamma)$, $i = 1,2$ and P is a polynomial whose angular boundary values satisfy almost everywhere on Γ the condition

$$\phi^+(t) = G(t)\phi^-(t) + g(t). \tag{5.2}$$

Systematic exposition of the theory of boundary value problems for analytic functions is given in the monograph [4]. In the case, where as admissible unknown functions we consider the functions representable by the Cauchy type integral with density from $L_p(\Gamma)$, $p > 1$ the problem has been studied in [5].

By Theorem 4, the class of unknown functions consists in our case of the Cauchy type $\widetilde{L}$-integrals (4.1) with the polynomial principal part at infinity (see also [2]). One might restrict this class by the Cauchy-Lebesgue type integrals (1.1), but nevertheless, as we shall see below, there will arise the question whether it is advantageous (or sometimes necessary) to extend it to the class of functions considered in the above-formulated problem (even for more rigid conditions imposed on g).

Generally speaking, this problem has no solution in the class of the Cauchy-Lebesgue type integrals even under the simplest assumptions with respect to Γ and G, what we can easily see by using the following example: Γ is the unit circumference, $G \equiv -1$, and $g \in L(\Gamma)$, for which $S_\Gamma(g) \notin L(\Gamma)$.

Suppose that there exits the extension $\phi(z) = K_\Gamma(\varphi)(z)$, $\varphi \in L(\Gamma)$, $z \notin \Gamma$. Then by the boundary condition (5.2) and formulas (1.2), $\phi^+(t) + \phi^-(t) = g(t) = S_\Gamma(\varphi)(t)$. Using the inversion formula for the singular Cauchy integral (see, for e.g., [2]), we obtain $\varphi(t) = S_\Gamma(g)(t)$, which is impossible, since $\varphi \in L(\Gamma)$, $S_\Gamma(g) \notin L(\Gamma)$.

A solution will be sought in a more wide class of the Cauchy type $\widetilde{L}$-integrals with the polynomial principal part at infinity, i.e., in a class of functions representable in terms of (5.1).

If we consider a particular case, where $G(t) \equiv 1$, i.e., the boundary condition (4.2) has the form

$$\Psi^+(t) - \Psi^-(t) = g(t), \tag{5.3}$$

then on the basis of equality (4.2) we can conclude that a solution will be given by the formula

$$\Psi(z) = K_\Gamma(g)(z) + P(z) \tag{5.4}$$

(that is by the Cauchy-Lebesgue type integral). We can easily see that (5.4) provides us with all the solutions of problem (5.3). Indeed, if we assume the existence of the other solution (in the class of $\widetilde{L}$-integrable functions)

$$\Psi_1(z) = \frac{1}{2\pi i}(\widetilde{L}) \int_\Gamma \frac{\psi_1(t)}{t-z} dt + P_1(z),$$

where $\psi_1 \in \widetilde{L}(\gamma)$, then owing to (4.2) and (5.3), we obtain $\Psi_1^+(t) - \Psi_1^-(t) = \psi_1(t) = g(t)$. Hence, $\Psi_1 = \Psi$.

The boundary condition (5.2) in the general case reduces to the condition (see, for e.g., [4], Ch. II, §37)

$$\frac{\phi^+(t)}{\chi^+(t)} - \frac{\phi^-(t)}{\chi^-(t)} = \frac{g(t)}{\chi^+(t)}, \tag{5.5}$$

where χ^+ and χ^- are the boundary functions of the so-called canonical solution $\chi(z)$ of the homogeneous boundary value problem $\phi^+(t) = G(t)\phi^-(t)$ (see [4], Ch. II, §35).

$\chi(z)$ is characterized by the following conditions: at a finite distance it never vanishes, is representable in terms of $\chi(z) = K_\Gamma(x)(z) + Q(z)$, where $\chi \in H_\Gamma(\alpha)$, and Q is a well-defined polynomial; the functions χ^+ and χ^- belong to $H_\Gamma(\alpha)$ (see, for e.g., [6], p. 253) and do not vanish (hence $(\chi^+)^{-1}$ and $(\chi^-)^{-1}$ also belong to $H_\Gamma(\alpha)$) and, because of the remark to Theorem 1, satisfy condition (3.1).

Since Γ is the Carleson curve the functions $(\chi^+)^{-1}$ and $(\chi^-)^{-1}$ satisfy condition (3.1). By Theorems 1 and 5 $(\chi^+)^{-1}\phi^+$ and $(\chi^-)^{-1}\phi^-$ belong to $\widetilde{L}(\Gamma)$ and are the boundary functions of the Cauchy-Lebesgue type integrals in D^+ and D^-, respectively.

Thus we have arrived at the analogous to (5.3) problem (5.4) (for the Cauchy type $\widetilde{L}$-integrals), whose general solution is given by the formula

$$\Psi(z) = \phi(z)/\chi(z) = \frac{1}{2\pi i} \int_\Gamma \frac{g(t)dt}{\chi^+(t)(t-z)} + P(z). \tag{5.6}$$

The general solution ϕ of the above-formulated problem (5.2) is given by equality (5.5) which under the above-imposed assumptions is representable by Theorem 5 in D^+ and D^- by the Cauchy-Lebesgue type integrals, i.e., in $D^+ \cup D^-$ by the Cauchy type $\widetilde{L}$-integral (with the polynomial principal part at infinity).

In particular, from formula (5.5) it follows that:

(1) if g is summable together with $S_\Gamma(g)$, then any solution of the problem under consideration is representable in $D^+ \cup D^-$ by the Cauchy-Lebesgue type integral;

(2) if $g \in L_p(\Gamma)$, $p > 1$ then any solution of the problem is representable in $D^+ \cup D^-$ by the Cauchy type integral with density from $L_p(\Gamma)$;

(3) if $g \in H_\Gamma(\alpha)$, then any solution of the problem is representable in $D^+ \cup D^-$ by the Cauchy type integral with density from $H_\Gamma(\alpha)$.

Thus in cases (1), (2) and (3) the extension of the class of unknown functions to the class of the Cauchy type $\widetilde{L}$-integrals fails to give new solutions.

Remark 1. As we see from the course of the proof, the function g can be taken from the class $\widetilde{L}(\Gamma)$ (replacing the Lebesgue integral in formula (5.5) by the $\widetilde{L}$-integral).

Remark 2. Throughout the paper, the line of integration Γ is supposed to be closed curve satisfying certain conditions. But all the results are also valid when the line of integration Γ consists of a finite number of mutually intersecting curves of the kind indicated above (if the domains D^+ and D^- are defined appropriately).

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