Proc. A. Razmadze Math. Inst. 117(1998), 139–140

L. Ephremidze

ON REVERSE WEAK $(1,1)$ TYPE INEQUALITIES FOR THE MAXIMAL OPERATORS WITH RESPECT TO AN ARBITRARY MEASURE

Let ν be a locally finite positive Borel measure on the real line $\mathbb{R}$, and let M_ν and M_ν^+ be maximal operators with respect to ν, i.e.,

$$M_\nu^+(f)(x) = \sup_{x<b,\nu[x,b)>0} \frac{1}{\nu[x,b)} \int\limits_{[x,b)} |f|\,d\nu, \tag{1}$$

$$M_\nu(f)(x) = \sup_{a<x<b,\nu(a,b)>0} \frac{1}{\nu(a,b)} \int\limits_{(a,b)} |f|\,d\nu, \tag{2}$$

$f \in L_{loc}(d\nu)$, $x \in \mathbb{R}$.

An operator $T : L_{loc}(d\nu) \to L_0(\mathbb{R})$ (the latter notation stands for the class of measurable functions) is said to be of reverse weak (1,1) type if there exists an independent constant C such that

$$\nu\Big\{(T(\chi_I f) > \lambda) \cap I\Big\} \geq \frac{1}{\lambda \cdot C} \int\limits_{\{(T(\chi_I f)>\lambda)\cap I\}} |f|\,d\nu,$$

for every $f \in L_{loc}(d\nu)$ and $I=(a,b)$, whenever $\lambda > \max\Big(T(\chi_I f)(a), T(\chi_I f)(b)\Big)$.

That the maximal operators M_ν and M_ν^+ are of reverse weak (1,1) type when ν is the Lebesgue measure was proved in [1], [2]. It is also well-known that in general M_ν and M_ν^+ may not be of this type. Theorems 1 and 2 below give necessary and sufficient conditions on the measure ν for the corresponding maximal operators (1) and (2) to be of reverse (1,1) type.

Theorem 1. *There exists a constant C such that*

$$\nu\Big\{(M_\nu^+(\chi_I f) > \lambda) \cap I\Big\} \geq \frac{1}{\lambda \cdot C} \int\limits_{\{(M_\nu^+(\chi_I f)>\lambda)\cap I\}} |f|\,d\nu$$

for every $f \in L_{loc}(d\nu)$ and $I=(a,b)$ whenever $\lambda > M_\nu^+(\chi_I f)(a)$ if and only if

$$\sup_{\nu(a,b)>0} \frac{\nu[a,b)}{\nu(a,b)} \leq C.$$

Theorem 1 asserts that if ν has some atom and M_ν^+ is of reverse weak (1,1) type, then starting from this point ν necesserily consists of isolated atoms.

1991 *Mathematics Subject Classification.* 42B25.
Key words and phrases. Maximal functions, reverse weak type inequality.

Theorem 2. *There exists a constant C such that*

$$\nu\Big\{(M_\nu(\chi_I f) > \lambda) \cap I\Big\} \geq \frac{1}{\lambda \cdot C} \int\limits_{\{(M_\nu(\chi_I f)>\lambda)\cap I\}} |f| \, d\nu$$

for every $f \in L_{loc}(d\nu)$ and $I = (a,b)$ whenever $\lambda > \max(M_\nu(\chi_I f)(a), M_\nu(\chi_I f)(b))$ if and only if

$$\sup_{\nu(a,b)>0} \min\left(\frac{\nu[a,b)}{\nu(a,b)}, \frac{\nu(a,b]}{\nu(a,b)}\right) \leq C.$$

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Authors' address:
A. Razmadze Mathematical Institute
Georgian Academy of Sciences
1, M. Aleksidze St., Tbilisi 380093
Georgia