Proc. A. Razmadze Math. Inst. 117(1998), 160–161

Sh. Tetunashvili

ON SOME PROPERTIES OF MULTIPLE TRIGONOMETRIC SERIES

Let $A \subset [0,1]$ and $B \subset [0,1]$. The set $A \times B$ is Cartesian product of sets A and B; i and j are non-negative integer numbers. Let μA be Lebesgue measure of A. The symbol $(i,j) \geq (0,0)$ means that $i \geq 0$ and $j \geq 0$, and $(i,j) \geq (1,1)$ means that $i \geq 1$ and $j \geq 1$. We denote the trigonometric system defined on $[0,1]$ by $T = \{t_i(\tau)\}_{j=0}^{\infty}$, where $t_0(\tau) \equiv 1$, $t_{2j-1}(\tau) = \sqrt{2}\cos 2\pi i\tau$, $t_{2i}(\tau) = \sqrt{2}\sin 2\pi i\tau$; $i = 1, 2, 3, \ldots$.

Consider trigonometric series

$$\sum_{i=0}^{\infty} a_i t_i(\tau). \tag{1}$$

Definition 1. A measurable set $A \subset [0,1]$ belongs to the class $U(T)$ if the convergence of (1) on A to zero implies $a_i = 0$ for any i.

It is well known, that $U(T) \neq \emptyset$ and if $A \in U(T)$, then $\mu A = 1$ (see[1], Chapter 9, Paragraph 3).

Consider double trigonometric series

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{ij} t_i(x) t_j(y); \quad (x,y) \in [0,1]^2. \tag{2}$$

The convergence of the series (2) will be understood as Pringsheim convergence. The equality

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{ij} t_i(x) t_j(y) = S$$

means that the series (2) is convergent to the number S in the point (x,y) in Pringsheim sense.

In the sequel we shall assume that $A \in U(T)$ and $B \in U(T)$; Functions $f(\tau)$ and $g(\tau)$ are finite functions defined on $[0,1]$.

We have the following results:

Theorem 1. *Let*

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{ij} t_i(x) t_j(y) = f(x) \cdot g(y) \tag{3}$$

when $(x,y) \in A \times B$.

Then there exist sequences of numbers $\{\alpha_i\}_{i=0}^{\infty}$ *and* $\{\beta_j\}_{j=0}^{\infty}$ *such that* $a_{ij} = \alpha_i \cdot \beta_j$ *for each* $(i,j) \geq (0,0)$.

1991 *Mathematics Subject Classification.* 42B05, 42C05.

Key words and phrases. Double trigonometric series, multiple trigonometric series, convergence in Pringsheim sense.

Theorem 2. *Let*

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{ij} t_i(x) t_j(y) = f(x) + g(y) \tag{4}$$

when $(x, y) \in A \times B$.

Then $a_{ij} = 0$ *for each* $(i, j) \geq (1, 1)$.

Theorem 3. *Let the series* (2) *converge on* $A \times A$ *to a finite function* $F(x, y)$, *for which*

$$F(x, y) = F(y, x) \tag{5}$$

when $(x, y) \in A \times A$.

Then $a_{ij} = a_{ji}$ *for each* $(i, j) \geq (0, 0)$.

Analogous theorems are valid also for d-fold multiple ($d \geq 3$) trigonometric series.

Remark 1. Theorems 1, 2 and 3 are best possible in the following sense: there exist $f(\tau)$ and $g(\tau)$ finite functions on $[0, 1]$, $F(x, y)$ finite function on $[0, 1]^2$, set $C \subset [0, 1]$ with $\mu C = 1$ such that relations (3), (4) and (5) hold for some series (2) when $(x, y) \in C \times C$, but Theorems 1, 2 and 3 are not true.

References

1. A. Zygmund, Trigonometric series, V. 1, *Cambrige Univ. Press, Cambrige*, 1959

Author's address:
Georgian Technical University
77, Kostava St., Tbilisi 380075
Georgia