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M. Okropiridze

RADIAL LIMITS OF DERIVATIVES OF THE SPHERICAL POISSON INTEGRAL

Let, the function $f(\theta,\phi)$ is summable on the rectangle $R=\{\theta,\phi): 0\le\theta\le\pi,\ -\pi\le\phi\le\pi\}$ and is 2π -periodic with respect to ϕ for all values of θ from $[0,\pi]$. The spherical Poisson integral with density $f(\theta,\phi)$ is denoted by the symbol $U_f(r,\theta,\phi)$:

$$U_f(r,\theta,\phi)=\frac{1}{4\pi}\int\limits_0^\pi\int\limits_{-\pi}^\pi f(\theta,',\phi')P_r(\theta,\phi;\theta',\phi')\sin\theta'\,d\theta'\,d\phi',$$

where

$$P_r(\theta,\phi;\theta',\phi')=(1-r^2)/G_r^{3/2}(\theta,\phi;\theta',\phi'),$$
$$G(\theta,\phi;\theta',\phi')=1-2r\Big[\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\phi'-\phi)\Big]+r^2,\quad 0\le r<1.$$

In the present note we investigate the existence of radial limits of derivatives $\frac{\partial}{\partial\theta}U_f$ and $\frac{\partial}{\partial\phi}U_f$. We use the notion of symmetric partial derivative in the strong sense [1], also the notion of right and left partial derivatives in the strong sense [2].

Definition 1. A function $f(x,y)$ is said to possess at (x_0,y_0) symmetric partial derivative in the strong sense with respect to x (with respect to y), if there exists the limit

$$\lim_{(h,k)\to(0,0)}\frac{f(x_0+h,y_0+k)-f(x_0-h,y_0+k)}{2h}\equiv f_{[1]}^{(')}(x_0,y_0)$$

$$\Big(correspondingly,\ \lim_{(h,k)\to(0,0)}\frac{f(x_0+h,y_0+k)-f(x_0+h,y_0-k)}{2k}\equiv f_{[2]}^{(')}(x_0,y_0)\Big)$$

Definition 2. A function $f(x,y)$ is said to possess at (x_0,y_0) right (left) partial derivative in the strong sense with respect to x if there exists the limit

$$\lim_{h\to0+,k\to0}\frac{f(x_0+h,y_0+k)-f(x_0,y_0+k)}{h}\equiv f_{[1]}^{+}(x_0,y_0)$$

$$\Big(correspondingly,\ \lim_{h\to0-,k\to0}\frac{f(x_0+h,y_0+k)-f(x_0,y_0+k)}{h}\equiv f_{[1]}^{-}(x_0,y_0)\Big)$$

This just is analogous with respect to y:

$$f_{[2]}^{+}(x_0,y_0)\equiv\lim_{k\to0+,h\to0}\frac{f(x_0+h,y_0+k)-f(x_0+h,y_0)}{k},$$

$$f_{[2]}^{-}(x_0,y_0)\equiv\lim_{k\to0-,h\to0}\frac{f(x_0+h,y_0+k)-f(x_0+h,y_0)}{k}$$

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Theorem 1. *If the function* $f(\theta,\phi)$ *possesses at a point* (θ_0,ϕ_0), $0<\theta_0<\pi$, $-\pi\le\phi_0\le\pi$, *finite symmetric partial derivative in the strong sense with respect to* ϕ, *in symbols* $f_{[2]}^{(')}(\theta_0,\phi_0)$, *then the equality*

$$\lim_{r\to 1}\frac{\partial}{\partial\phi}U_f(r,\theta_0,\phi_0)=f_{[2]}^{(')}(\theta_0,\phi_0)$$

is fulfilled.

Theorem 2. *If the function* $f(\theta,\phi)$ *possesses at a point* (θ_0,ϕ_0), $0<\theta_0<\pi$, $-\pi\le\phi_0\le\pi$, *finite left and finite right partial derivatives in the strong sense, in symbols* $f_{[1]}^{-}(\theta_0,\phi_0)$, $f_{[1]}^{+}(\theta_0,\phi_0)$, *and there exists finite partial limit* $\lim\limits_{\phi'\to\phi}f_0(\theta_0,\phi)$, *then the equality*

$$\lim_{r\to 1}\frac{\partial}{\partial\Theta}U_f(r,\theta_0,\phi_0)=\frac{1}{2}\left[f_{[1]}^{-}(\theta_0,\phi_0)+f_{[1]}^{+}(\theta_0,\phi_0)\right]=f_{[1]}^{(')}(\theta_0,\phi_0)$$

is fulfilled.

Remark 1. The cases of values $\theta_0=0$ and $\theta_0=\pi$ is investigated in [3].

References

1. M. Okropiridze, Symmetrically differentiable functions of two variables (*to appear*).
2. O. Dzagnidze, Unilateral continuity and unilateral partial derivatives in various senses (*to appear*).
3. O. Dzagnidze, On angular limits at the poles of a sphere of the derivatives of the Poisson integral. (Russian) *Proc. A. Razmadze Math. Inst.* **98**(1990), 99–111.

Author's address
Secondary School No. 51
2, Melikishvili St., Tbilisi 380057
Georgia