Proc. A. Razmadze Math. Inst. 112(1997), 146–147

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ON NON-OSCILLATORY DIFFERENTIAL EQUATIONS

(Reported on November 14, 1996)

Let us consider a linear differential equation

$$L(u)=0, \tag{1}$$

where

$$L(u)=(-1)^m(wu^{(m)})^{(m)}+qu.$$

We assume that q is a locally summable on (a,∞) function, $w^{(k)}\in AC$, $k=1,2,\dots,m-1$, on every finite segment and a non-negative function w is such that $\frac{1}{w}$ is summable on (a,∞).

Differential equation (1) is said to be oscillatory at infinity if for every $t>a$ there is a solution of equation (1) having to the right of the point t more than one n-fold zero. Equation (1) is called non-oscillatory, otherwise (see [1]).

In this note we present some sufficient conditions for equation (1) to be non-oscillatory. For ease of description we consider the case $m=1$.

The following assertions are valid.

Theorem 1. *If there exists a number $s>1$ such that*

$$\int_a^\infty |q(t)|^s w^{s-1}\bigg(\int_t^\infty \frac{d\tau}{w(\tau)}\bigg)^{2s-1}dt<\infty,$$

then equation (1) is non-oscillatory.

Theorem 2. *If there exist a number $s>1$ and a locally summable function v such that the conditions*

$$\sup_{x>a}\bigg(\int_a^x v^{s'}(t)dt\bigg)^{\frac{1}{s'}}\bigg(\int_x^\infty \frac{dt}{w(t)}\bigg)<\infty,\quad s'=\frac{s}{s-1},$$

and

$$\int_a^\infty |q(t)|^s v^{-s}(t)dt<\infty,$$

are fulfilled, then equation (1) is non-oscillatory for $m=1$.

1991 *Mathematics Subject Classification.* 34C10.

Key words and phrases. Linear differential equation, non-oscillatory equation, Lorentz space.

From the latter theorem we can derive Theorem 1 by substituting

$$v(t) = w^{-\frac{1}{s'}} \left(\int_t^\infty \frac{d\tau}{w(\tau)} \right)^{-1-\frac{1}{s'}} dt < \infty.$$

Corollary 1. *For equation* (1) *to be non-oscillatory, it suffices for* $m = 1$, $w(x) = x^{1+\varepsilon}$, $\varepsilon > 0$, *to fulfil the condition*

$$\int_a^\infty |q(t)|^s t^{s-s\varepsilon-1} dt < \infty$$

for any $s > 1$.

Corollary 2. *For* $w(x) = x \ln^\gamma x$, $\gamma > 1$, *the condition*

$$\int_a^\infty |q(t)|^s t^{s-1} \ln^{-\gamma s+2s-1} t < \infty$$

for any $s > 1$ *is sufficient for equation* (1) *to be non-oscillatory.*

Note that the class of differential equations in the above-cited theorems can be widened by using more fine scale of spaces than that of the Lebesgue spaces.

Let L^{sr}, $(1 < s,\, r < \infty)$ be a Lorentz space whose norm is defined as follows:

$$\|f\|_{L^{sr}} = \left(r \int_0^\infty \lambda^r \big|\{x : \, |f(x)| > \lambda\}\big|^{\frac{r}{s}} \frac{d\lambda}{\lambda} \right)^{\frac{1}{r}}.$$

Theorem 3. *If there exist numbers* $s > 1$ *and* $r > s$ *and a weight function* v *such that the conditions*

$$\sup_{t>0} \big\| v\chi_{\{x<t\}} \big\|_{L^{s'r'}} \int_t^\infty \frac{dx}{w(x)} < \infty$$

and

$$\|qv^{-1}\|_{L^{sr}} < \infty,$$

are fulfilled, then equation (1) *is non-oscillatory for* $m = 1$.

References

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