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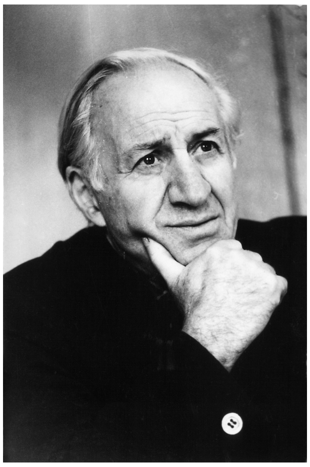

NICHOLAS (NIKO) VAKHANIA

V. KVARATSKHELIA AND V. TARIELADZE

1. Short biography

Nicholas (Niko) Vakhania was born on August 28, 1930 in Kutaisi (Georgia). His parents Tamar and Nicholas Vakhania had also an another son George[1]. Their father was a tradition-follower, hard-working and honest person. He gave to his both sons, among many other good qualities, the love of country, honesty, diligence; the trend towards knowledge they seemingly had genetically.

From 1931 Vakhania's family lived in the capital of Georgia, Tbilisi. Niko started his school-study in 1938 and left it in 1949. During difficult times of the Second World War he was often sent to his father's native village Khabume (Western Georgia); he even attended the local school there over

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[1]George Vakhania (1931-1991) was an experienced engineer-metallurgist.

several months period. In 1949 he entered Tbilisi State University (TSU), physics department.

His distinguished abilities and results in study were noticed; he was considered as one of the brightest students and from the third year of studying he had been receiving a special scholarship. During his university study he understood that his main interest was Mathematics, it was the Mathematics what was important for him. And when he graduated from the physics department in 1954, he began his postgraduate study in Tbilisi A. Razmadze Mathematical Institute.

He was recommended by academician[2] Niko Muskhelishvili to continue his postgraduate course with academician S. Sobolev at Moscow Lomonosov State University (MSU), Department of Mathematics and Mechanics. N. Muskhelishvili even provided him with a letter of recommendation to S. Sobolev. For N. Vakhania the years spent at MSU were really fruitful and interesting. Many young fellows, who later became famous mathematicians (among them A. Skorokhod, V. Sazonov, V. Arnold, B. Boyarski etc.) had their course of study at the same time. N. Vakhania kept close and friendly relationship with them till the end of his life.

In 1958 at the scientific defense council of Department of MSU Mathematics and Mechanics N. Vakhania successfully defended his candidate dissertation: “On some boundary values problems for string vibration equation on rectangular domains”.

In 1956 Computing Center of Academy of Sciences of Georgian SSR was founded, where in 1957 N. Vakhania became a senior scientific worker. From 1961 he was a head of the Department of Mathematical Cybernetics (later, the Department of Probability Theory and Functional Analysis) of the Computing Center. At the same time he began his long career of teaching at TSU (at the Department of Mechanics and Mathematics and later, at the department of Cybernetics as well). During this period he became as professional lecturer in mathematics. He was giving his lectures and talks with amazing zeal and enthusiasm. His lectures were for the most part exceedingly lucid and stimulating; from time to time he would indicate where further investigation was badly needed. He was trying and was succeeding to make his listeners to follow him, to think together with him, to make from them the participants of the process. He was a serious and concentrated listener as well. The colleagues and students will forever remember his exceptional style of exposition in fine and convincing way, which, with years, was becoming more and more perfect. N. Vakhania had no language barriers – he, together with the native Georgian, knew well Russian and English. Everything this was a result of his intensive hard work, which was

[2]Here and in what follows “academician” will mean “a member of the academy of sciences of USSR”.

customary for him from his childhood; the similar relation to work he was requiring from his students. He often used to say that Mathematics is like a capricious lady, who does not forgive betrayal.

In 1969 at the scientific defense council of Mathematics and Mechanics department of TSU N. Vakhania successfully defended his doctoral dissertation: "Some questions of the theory of probability distributions in linear spaces". In 1970-1973 he was a dean Cybernetics and Applied Mathematics Department of TSU, in 1971-2004 he occupied a chair of Theory of Random Processes of TSU.

The creative activities of N. Vakhania were not restricted only to Science and Teaching. His talent of organization had manifested itself clearly and completely during the period of his work as a director of the Institute of Computational Mathematics (ICM), which continued for 27 years from 1978. He participated actively in the work of the section of Mathematics and Physics of Georgian Academy of Sciences, where during many years he was a vice academic-secretary. N. Vakhania took part in many scientific boards, such as "Encyclopaedia of Probability Theory and Mathematical Statistics" (Moscow), "Georgian Encyclopaedia" (Tbilisi), the series of Stochastics of the Publishing House PHASIS (Moscow), journal "Probability Theory and its Applications" (Moscow), "Georgian Mathematical Journal" (Tbilisi). He was a member of International Coordination Committee for Computational Mathematics and a member of the International Fund for Innovation.

In 2005 N. Vakhania, at his own will, left the position of the director of ICM and till the end of his life he was a chief scientific worker of the Institute. In 2008-2013 he was a had of the Scientific Council of the Institute.

N. Vakhania died on July 23, 2014, at the age of eighty-four.

Although the administrative activities were taking rather considerable time of N. Vakhania, he was nevertheless continuing the research work. His first scientific papers were dedicated to the theory of differential equations. He studied non-classical boundary value problems for the hyperbolic equations, the problem of small vibrations of a top filled by a liquid, etc. The results obtained by him in this direction are still of great interest.

The next cycle of scientific works of N. Vakhania were about the Theory of Probability Distributions in Linear Spaces. He gave a description of Gaussian distributions in the classical sequence spaces and created the Covariance Theory of Probability Distributions in infinite dimensional spaces. His results obtained in this direction are collected in monographs: "Probability Distributions on Linear Spaces" (in Russian, Tbilisi, "Metzniereba", 1971; authorized English Edition, Amsterdam, "North Holland", 1981) and "Probability Distributions on Banach Spaces" (in Russian, Moscow, "Nauka" 1985; authorized English Edition, Dordrecht, D. Reidel PC, 1987, co-authors: V. Tarieladze and S. Chobanyan). N. Vakhania is the author of more than 80 scientific works, which are devoted to the theory of differential equations, modern and classical Probability Theory, Functional Analysis, Computational Mathematics and quaternionic Probability Theory.

In Georgia N. Vakhania has founded a new direction of mathematics – Probability Theory in Infinite-Dimensional Linear Spaces. His fruitful scientific activity had a significant influence on the development of this direction of research. His classical monographs, as many foreign colleagues underline, still have numerous citations and remain a source of new ideas. N. Vakhania was often invited to various scientific centers and universities of all continents of the world, made joint scientific researches with foreign colleagues, gave courses of lectures. His scientific activities abroad and in his native country indicate a wide international recognition of his talent and contributions. Among Soviet mathematicians he was one of the first, who was invited for long term scientific work to USA and Japan. Mathematicians were the men with whom he had worked all his life. His archive keeps more than 40 invitations from distinguished foreign mathematicians, all of which were realized.

The scientific school created by him soon received high authority in Georgia and abroad. During the years this School is recognized as one of the leading centers of Probability Theory in Linear Spaces. It will not be without an interest to reproduce here estimations of some famous scientists about N. Vakhania and his school.

Academician S. Sobolev: "N. Vakhania is a many-sided and deeply educated mathematician, who received important and elegant results in nonclassical boundary value problems of the theory of differential equations, in the theory of random processes, in applications of Functional Analysis to Probability Theory, in Functional Analysis itself".

Academicians A. Kolmogorov, I. Prokhorov, V. Pugachov and V. Mikhalevich: "The works of N. Vakhania and his pupils in the direction of the theory of probability distributions in functional spaces successfully can compete with the best works written in the same direction over the world. The contribution of N.Vakhania in the development of this field has received an international recognition".

Academician of Lithuanian Academy of Sciences I. Kubilius: "Professor N. Vakhania is one of the greatest specialist of the Theory of Probability in Functional Spaces in our country, his name is well-known far outside our country too".

Academician of Lithuanian Academy of Sciences V. Statuliavichus: "N. Vakhania is a leader and creator of an internationally recognized school of Probability Theory. The scientific contacts with this school played a role in the development of Probability Theory in Lithuania".

N. Vakhania supervised 12 candidate dissertations. Among his pupils are 4 doctors (Sergei Chobanyan, Nguyen Duy Tien, Vaja Tarieladze and

Vakhtang Kvaratskhelia). Training future researchers was one of his most important jobs.

The scientific and pedagogical contributions of N. Vakhania were well recognized and estimated. He was a member of the Georgian National Academy of Sciences, owner of Order of Honor; premiums of Andrea Razmadze and Niko Muskhelishvili of Georgian Academy of Sciences; he twice received an honorable award of TSU – Ivane Javakhishvili Medal; he had the title of Honorable Scientist. A special issue of "Georgian Mathematical Journal" was dedicated to 70th birthday of N. Vakhania (Vol. 8 (2001), Number 2).

Everybody who knew N. Vakhania, gave a high estimation to his professionalism, to his objectivity, to the fact that he was a man of principle; he was famous for his outstanding honesty and courage. N. Vakhania was with sound mathematical ambitions and a genuine passion for mathematics; for him mathematics was the one great permanent happiness of his life, it gave him great pleasure to work. Only research, teaching, writing and his family filled his days with pleasure. He was not only an eminent mathematician but also a gentle and kind man, eager to do everything in his power to help all with whom he came in contact.

N. Vakhania's wife Tsiala Maisuradze is an English teacher. Many mathematicians, former students of TSU are her pupils. Nicholas and Tsiala have two sons Zurab and Nodari. They became known scientists. Zurab is a candidate of Mathematics and doctor of Pedagogy, he is a member of Pedagogical Academy of Georgia, a vice-director of Uznadze Institute of Psychology, the author of many text-books. Nodari is a doctor of Mathematics and professor of Quernavaca University (Mexico).

Let us finish this short biographical account by N. Vakhania's own words directed to the young generation twenty years ago, and which have not lost their actuality until today: "A necessary condition to achieve a big success is to love the claim, to get enjoyment not only from an achieved success and its consequences, but also from the process of work. The work must be a permanent process. God distributes talent and abilities among people not uniformly, but no one receives them so much, which could guarantee to achieve a big success without a big effort".

The job to which the years of life of N. Vakhania was dedicated continues to be alive in Niko Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University.

2. Mathematical works

N. Vakhania loved Mathematics. He never published anything until it was as finished and perfect as he could make it. He used to say: mathematicians are makers of intellectual tools. For him Mathematics was Art. Careful research and accurate writing make his works a valuable reference tool. He was always to the point, thus reflecting the quality of precision that was also notable in his scientific work.

2.1. **Partial differential equations.** N. Vakhania in his candidate thesis (Moscow, MSU, 1958) studied boundary values problems for a string vibration equation on rectangular domains and obtained the result which turned out to be very interesting for the Moscow mathematical community.

The first problem studied by N. Vakhania can be formulated as follows:

Problem 1. Let L_1 and L_2 be arbitrary positive numbers. In the rectangle $R = [0, L_1] \times [0, L_2]$, find a continuous solution u for the string vibration equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

which satisfies the boundary condition

$$u|_\Gamma = f$$

where Γ is the boundary of the rectangle R, while f is the given continuous function defined on Γ.

Let $\rho = \frac{L_1}{L_2}$. N. Vakhania stated the following:

- If ρ is irrational, then Problem 1 has a unique continuous solution. Moreover, in this case the set of all boundary functions f for which Problem 1 has a continuous solution is uniformly dense in $C(\Gamma)$.
- If ρ is rational, then Problem 1 may have no continuous solution for the given continuous boundary functions f. Moreover, in this case the set of all boundary functions f for which Problem 1 has a continuous solution is not uniformly dense in $C(\Gamma)$.

To formulate one of the main results of N. Vakhania in this direction we need the following notation. Let $p = (p_0)$ be a point on Γ and for a fixed natural number i let p_i be a point, where the line $x + (-1)^i y = \text{const}$ joins p_{i-1} and Γ. Write:

$$S_N(p) = \sum_{k=0}^{2N-1} (-1)^k f(p_k), \quad T_N(p) = \sum_{k=0}^{2N-1} (-1)^{k+1} f(p_k)\,.$$

Theorem 1. *If ρ is irrational and f is continuous, then the necessary and sufficient condition for the existence and uniqueness of a solution of Problem 1 is the uniform $(C, 1)$-summability on Γ of sequences (S_N) and (T_N), when $N \to \infty$.*

The second problem studied by N. Vakhania is

Problem 2. In the rectangle R, find a continuous solution of the hyperbolic system

$$\frac{\partial u_1}{\partial x} = \frac{\partial u_2}{\partial y}, \quad \frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial x}$$

satisfying the boundary condition

$$au_1|_\Gamma + bu_2|_\Gamma = f\,,$$

where a, b and f are given continuous functions defined on Γ.

This problem for the case $\rho = 1$ has been considered earlier by S. Sobolev. N. Vakhania has shown that Sobolev's method can be applied to every rational ρ and that the obtained result in this case does not differ qualitatively from the case $\rho = 1$. For the case of an irrational ρ N. Vakhania established the uniqueness of a solution and showed that in some cases Problem 2 can be reduced to the Dirichlet Problem.

Theorem 2. *Suppose that the following three conditions are satisfied:*
(a) *There exists a constant A and a natural number n such that*

$$\left|\rho - \frac{m}{n}\right| > \frac{A}{n^r}$$

for some number $r > 0$ and for all (with the exclusion of a finite amount) pairs (m, n) of co-prime natural numbers,
(b) $\ln(a+b) \in C^{(2r+k,\varepsilon)}$, *where k is a non-negative integer and* $0 < \varepsilon < 1$,
(c) *The functions a and b satisfy the condition*

$$a^2 + b^2 \neq 0$$

at each point of Γ.

Then there exists a k-times continuously differentiable solution of Problem 2 for an arbitrary boundary function $f \in C^{(r+k,\varepsilon)}$.
Moreover, using generalized functions, N. Vakhania has constructed a space of functions such that the Dirichlet problem has a solution for every boundary function belonging to this space.

2.2. **Probability Theory.** The first probabilistic paper of N. Vakhania appeared in 1964, contains the following result:

Theorem 3. *Let $1 \leq p < +\infty$ and $p' = \frac{p}{p-1}$. A functional $\chi : l_{p'} \to \mathbb{C}$ is a characteristic functional of Gaussian distribution on l_p if and only if it has the form*

$$\chi(y) = \exp\left(i\sum_{k=1}^{\infty} m_k y_k + \frac{1}{2}\sum_{k,j=1}^{\infty} r_{k,j} y_k y_j\right), \quad \mathbf{y} = (y_k)_{k\in\mathbb{N}} \in l_{p'}\,,$$

where $\mathbf{m} = (m_k)_{k\in\mathbb{N}} \in l_p$ *and* $(r_{k,j})_{k,j=1}^{\infty}$ *is a positive definite matrix satisfying the condition*

$$\sum_{k=1}^{\infty} r_{k,k}^{p/2} < +\infty\,.$$

Previously, a similar statement was known only for the case $p = 2$ (*E. Mourier* (1953)).

In 1965 this result with a complete proof was published in C.R. Acad. Sci. Paris (the article was presented by P. Levy). Moreover, in C.R.'s article from Theorem 3 follows the following

Theorem 4. *Let* $1 \le p < +\infty$ *and* μ *be a Gaussian distribution on* l_p. *Then*

$$\int\limits_{l_p} \|x\|^t d\mu(x) < +\infty \quad \forall t \in [0, < +\infty[\,.$$

This was the first integrability result obtained for the case $p \neq 2$.

Another important result proved by N. Vakhania with the help of Theorem 3 was the central limit theorem for a sequence of independent identically distributed random elements with values in l_p, $1 \le p < 2$.

In the following cycle of papers N. Vakhania continued the study of probability distributions and random elements in general Banach spaces. In 1968, in the paper published in Studia Mathematica he found the sufficient condition for the existence of Pettis integral (i.e. the mathematical expectation) of weak second order random elements in a separable Banach space; this sufficient condition made it possible to show that there exists mathematical expectation of Gaussian random elements in a separable Banach space. In the papers "Covariance operator of a probability distribution in a Banach space" (Bull. Acad. Sci. of the Georgian SSR 51 (1968), No. 1, 35–40) and "On covariance of random elements in linear spaces" (Bull. Acad. Sci. of the Georgian SSR 53(1969), No. 1, 17–20) N. Vakhania introduced the general concept of a covariance operator of a weak second order probability distribution on the Banach space and posed the basic problems of description of classes of all covariance and all Gaussian covariance operators. Earlier the similar concepts were defined only for the strong second order probability distributions in the Hilbert space (Yu. V. Prokhorov, 1956) and for the Gaussian distribution (L. LeCam, 1958). In particular, N. Vakhania has shown that in the case of a separable reflexive Banach space the class of all covariance operators coincides with that of all symmetric positive operators acting from the dual space into the initial space. Later, in the joint with V. Tarieladze paper "Covariance operators of probability measures in locally convex spaces" (Teoriya Veroyatnostei i ee Primeneniya, 23(1978), No. 1, 3–26) they established that a similar statement is valid for all separable Frechet spaces.

In their joint paper “Estimate of the rate of convergence in the central limit theorem in Hilbert space” (Proc. Computing Center of Georgian Acad. Sci. 9(1969), 150–160) N. Vakhania and N. P. Kandelaki have found the first estimate of the rate of convergence on ellipsoids in the central limit theorem in an infinite-dimensional Hilbert space.

N. Vakhania and S. A. Chobanyan in their joint paper “Wide sense stationary processes with values in Banach spaces” (Bull. Acad. Sci. of the Georgian SSR 57(1970), No. 3, 545–548) initiated the study of wide sense stationary processes with values in infinite-dimensional Banach spaces; earlier, this kind of processes were treated only in one-dimensional case (A. Y. Khinchin–1934, H. Cramer–1940, I. A. Rozanov–1967), in a finite-dimensional case (N. Wiener – P. Masani–1957) and in an infinite-dimensional Hilbert space (R. Payen–1967).

In his paper “On subgaussian random vectors in normed spaces” (Bull. Georgian Acad. Sci., 163(2001), No. 1, 8–11) N. Vakhania has shown that the integrability results similar to that of Gaussian random vectors do not hold for subgaussian random vectors in infinite-dimensional normed spaces.

N. Vakhania with V. Kvaratskhelia in their paper “Unconditional convergence of weakly sub-gaussian series in Banach spaces” (Teoriya Veroyatnostei i ee Primeneniya 51.2 (2006): 295–318) have obtained the conditions of almost sure unconditional convergence of series of sub-gaussian random elements.

N. Vakhania’s last published probabilistic paper “Skitovich-Darmois theorem for complex and quaternion cases” (Proceedings of A. Razmadze Mathematical Institute 160(2012), 165–169) written jointly with G. Chelidze contains an adequate formulation and elementary real variable proof of Skitovich-Darmois theorem for complex and quaternion-valued random variables.

The last years of his life N. Vakhania was intensively working on Georgian text-book in Probability Theory, the already written parts of which will appear soon in print.

2.3. **Functional Analysis.** The first work devoted to the pure Functional Analysis is a joint with I. N. Kartsivadze article “A remark on the intersection of embedded closed sets” (Matematicheskie Zametki 3(1968), No. 2, 165–170) in which for each non-empty bounded subset F of a Banach space X they defined a number $\kappa(F) \in [0,1]$ as follows:

$$\kappa(F) = \sup_{x\in F} \frac{r_x(F)}{R_x(F)},$$

where for $x \in F$,

$$r_x(F) = \inf_{y\in X\setminus F} \|x-y\|, \quad R_x(F) = \sup_{y\in F} \|x-y\|$$

and it is noted that $\kappa(F) = 1$ iff the closure of F is a ball.

Theorem 5. Let $(F_n)_{n\in\mathbb{N}}$ be a sequence of closed bounded non-empty subsets of the Banach space X such that $F_n \supset F_{n+1}$, $n = 1, 2, \dots$ and

$$\limsup \kappa(F_n) > \frac{1}{2}\,.$$

Then

$$\bigcap_{n=1}^{\infty} F_n \neq \emptyset. \tag{1}$$

From this theorem it follows, in particular, that if $(F_n)_{n\in\mathbb{N}}$ is a sequence of closed balls of the Banach space X such that $F_n \supset F_{n+1}$, $n = 1, 2, \dots$, then (1) holds.

The above-mentioned article contains also the following result.

Theorem 6. Let X be the Banach space of all convergent sequences of real numbers and $\kappa \in [0, \frac{1}{2}[$. Then there exists a sequence $(F_n)_{n\in\mathbb{N}}$ of closed bounded *convex* non-empty subsets of X such that

$$F_n \supset F_{n+1},\ \ \kappa(F_n) = \kappa,\ \ n = 1, 2, \dots\,,$$

but

$$\bigcap_{n=1}^{\infty} F_n = \emptyset. \tag{2}$$

Later it was shown that for the Banach space X there may exist a sequence $(F_n)_{n\in\mathbb{N}}$ of closed bounded non-empty subsets of X such that

$$F_n \supset F_{n+1},\ n = 1, 2, \dots, \quad \limsup \kappa(F_n) = \frac{1}{2}\,,$$

for which (2) holds (G. Chelidze, Bull. Georgian Acad. Sci., 156, No. 2, (1997), 207–209).

In his work "Orthogonal random vectors and the Hurwitz-Radon-Eckmann theorem" (Georgian Math. J. 1(1994), No. 1, 99–113) N. Vakhania has formulated and solved the following *generalized Hurwitz-Radon-Eckmann problem:* let H be a finite or an infinite-dimensional Hilbert space and $B : H \to H$ be a self-adjoint operator. Find a number of elements of a maximal (with respect to the set-theoretic inclusion) set $\mathcal{U}$ of orthogonal operators $U : H \to H$ with the following properties:

- $U^2 = -I$ for every $U \in \mathcal{U}$ (where I stands for the identity operator) and
$$U_1, U_2 \in \mathcal{U},\ U_1 \neq U_2 \implies U_1U_2 = -U_2U_1\,,$$
- $UB = BU$ for every $U \in \mathcal{U}$.

The questions dealing with the unconditional convergence of series in Banach spaces were treated in a cycle of papers written by N. Vakhania jointly with V. Kvaratskhelia. They are: "Absolute and unconditional convergence in l_1" (Bull. Georgian Acad. Sci., 160, No. 2, (1999), 201–203); "On a criterion for unconditional convergence of Hadamard series in the spaces $l_p, 1 \leq p < \infty$" (Bull. Georgian Acad. Sci., 162(2000), No. 2, 199–202); "An application of the Brunel-Sucheston spreading model" (Bull. Georgian Acad. Sci., 165(2002), No. 3, 453–457); "On unconditional convergence of series in Banach spaces with unconditional bases" (Bull. Georgian Acad. Sci. (New Series), 3 (2009), No. 1, 20–24).

3. Publications: Monographs

1. Probability Distributions on Linear Spaces. (Russian) *Metsniereba, Tbilisi*, 1971, 156 p.

2. Probability Distributions on Linear Spaces. (English Edition) North Holland Series in Probability and Applied Mathematics. New York, Oxford: North Holland. XIV, 123 p.

3. Probability distributions in Banach spaces. (Russian) *Nauka, Moscow*, 1985, 368 pp. (with V. Tarieladze and S. Chobanyan).

4. Probability distributions on Banach spaces. Translated from the Russian and with a preface by Wojbor A. Woyczynski. Mathematics and its Applications (Soviet Series), 14. *D. Reidel Publishing Co., Dordrecht*, 1987. (with V. Tarieladze and S. Chobanyan)

4. Publications: Scientific Articles

1. A boundary problem for a hyperbolic system equivalent to the string vibration equation. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **116** (1957), 906–909.
2. The Dirichlet problem for the vibrating string. (Russian) *Soobšč. Akad. Nauk Gruzin. SSR* **21** (1958), 131–138.
3. On the approximate solution of the Dirichlet problem for the equation of the string. (Russian) *Trudy Vyčisl. Centra Akad. Nauk Gruzin. SSR* **1** (1960), 41–49.
4. A special problem for an equation of mixed type. (Georgian) *Trudy Vyčisl. Centra Akad. Nauk Gruzin. SSR* **3** (1963), 69–80.
5. On normal distributions in l_p spaces. (Russian) *Teor. Verojatnost. i Primenen* **9** (1964) 737–738.
6. On the stability of spinning of a top with a liquid filled cavity. *Prikl. Mat. Meh.* **29** 35–45 (Russian); *translated as J. Appl. Math. Mech.* **29** 1965 36–47 (1966).
7. Characteristic functional for random sequences. (Russian) In "Some problems in the theory of stochastic processes", *Proc. Comp. Center Acad. Sci. Georgian SSR* **5** (1965), No. 1, 5–32.
8. Sur une propriété des répartitions normales de probabilités dans les espaces l_p, $(1 \leq p < \infty)$ et H. (French) *C. R. Acad. Sci. Paris* **260** (1965), 1334–1336.
9. Sur les repartitions des probabilites dans les espaces de suites numeriques, Comptes Rendus Acad. Sci. Paris, 260(1965), No. 6, 1560–1562.
10. On non-degenerate probability distributions in the space $l_p(l \leq p \leq \infty)$. (Russian) *Teor. Verojatnost. i Primenen.* **11** (1966), No. 3, 524–528.
11. A stochastic integral for operator-valued functions. (Russian) *Teor. Verojatnost. i Primenen* **12** (1967), 582–585 (with N. Kandelaki).
12. A probabilistic problem for the one-dimensional heat equation. (Russian) *Teor. Verojatnost. i Primenen.* **12** (1967), 727–729.

13. A remark on the intersection of a sequence of imbedded closed sets. (Russian) *Mat. Zametki* **3** (1968), 165–170 (with B. Kartsivadze).
14. A certain condition for the existence of the Pettis integral. (Russian) *Studia Math.* **29** (1968), 243–248.
15. Estimate of the speed of convergence in the multidimensional central limit theorem. (Russian) *Sakharth. SSR Mecn. Akad. Moambe* **50** (1968), 273–276 (with N. Kandelaki).
16. Distribution of a scalar product of Gaussian random vectors. (Russian) *Sakharth. SSR Mecn. Akad. Moambe* **50** (1968), No. 3, 535–540 (with N. Kandelaki).
17. The covariance operator of a probability distribution in a Banach space. (Russian) *Sakharth. SSR Mecn. Akad. Moambe* **51** (1968), 35–40.
18. On the covariance of random elements in linear spaces. (Russian) *Sakharth. SSR Mecn. Akad. Moambe* **53** (1969), 17–20.
19. An estimate of the rate of convergence in the central limit theorem in Hilbert space. (Russian) *Sakharth. SSR Mecn. Akad. Gamothvl. Centr. Šrom.* **9** (1969), No. 1, 150–160 (with N. Kandelaki).
20. A certain question on probability distributions in a Banach space. (Russian) *Gamoqeneb. Math. Inst. Sem. Mohsen. Anotacie.* **1** (1969), 37–39.
21. The covariation of a stochastic element in a Banach space. (Russian) *Trudy I. Vekua Inst. Appl. Math. Tbilisi State Univ.* **2** (1969), 179–184.
22. Processes stationary in the broad sense with values in a Banach space. (Russian) *Sakharth. SSR Mecn. Akad. Moambe* **57** (1970), 545–548 (with S. Chobanyan).
23. Gaussian random sequences. (Russian) *Tbilis. Univ. Šrom. A* **1(137)** (1971), 103–109.
24. On a property of Gaussian distributions in Banach spaces. *Journal of Indian statistical society, Sankhya Ser. A* **35** (1973), 23–28.
25. Träger des Gausschen Masses im Hilbertraum. (German) *Math. Nachr.* **64** (1974), 319–322.
26. The topological support of Gaussian measure in Banach space. *Nagoya Math. J.* **57** (1975), 59–63.
27. On the support of Gaussian probability measures on locally convex topological vector spaces. *Multivariate analysis, IV (Proc. Fourth Internat. Sympos., Dayton, Ohio, 1975)*, pp. 297–309. *North-Holland, Amsterdam*, 1977 (with B. Rajput).
28. Prolongements d'une dualité (X, Y) entre espaces vectoriels en une dualité de type (L^q, L^p). (French) *C. R. Acad. Sci. Paris Sér. A-B* **284** (1977), No. 24, A1561–A1564 (with V. I. Tarieladze and A. Tortrat).
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