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ON THE BOUNDARY VALUE PROBLEM OF FINDING TWO FUNCTIONS FOR A PLANE WITH LINEAR CUTS

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Abstract. For a plane with rectilinear cuts lying along the segments of two mutually perpendicular straight lines we consider the problem on finding two piecewise holomorphic functions when in the boundary conditions for the cuts to the given value of the first function is added the conjugate value of the second one.

The solution of the problem is constructed in two modes in the classes of N.Muskhelishvili, explicitly (in a closed form), by the Cauchy type integrals.

რეზიუმე. უსასრულო სიბრტყისათვის სწორხაზოვანი ჭრილებით, რომლებიც განლაგებულია ორი ურთიერთმართობი წრფის მონაკვეთების გასწვრივ, განხილულია ორი უბნობრივ ჰოლომორფული ფუნქციის პოვნის ამოცანა, როცა საზღვარზე მოცემულ ერთ-ერთი ფუნქციის მნიშვნელობას ემატება მეორის შეუღლებული მნიშვნელობა.

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Assume that the plane of a complex variable $z = x + iy$ is cut along $2p$ segments $a_k \le x \le b_k$, $-b_k \le x \le -a_k$, $k = 1, 2, \dots, p$ of the real axis and along $2m$ segments $\alpha_j \le y \le \beta_j$, $-\beta_j \le y \le -\alpha_j$, $j = 1, 2, \dots, m$ of an imaginary axis. Let L_k and L_{-k} be the cuts lying along the segments $[a_k, b_k]$ and $[-b_k, -a_k]$, and let Λ_j and Λ_{-j} be the cuts lying along the segments $[i\alpha_j, i\beta_j]$ and $[-i\beta_j, -i\alpha_j]$; L and Λ are unions of these cuts, respectively. A multiply connected domain, i.e., a plane with cuts, we denote by S.

Let us consider the boundary value problem: *Find functions $\varphi(z)$ and $\psi(z)$, piecewise holomorphic in S, satisfying the boundary conditions*

$$\big[\varphi(t) + \overline{\psi(t)}\big]^{\pm} = f^{\pm}(t), \quad t \in L \tag{1}$$

and

$$\big[\varphi(t) + \overline{\psi(t)}\big]_{\pm} = g_{\pm}(t), \quad t \in L, \tag{2}$$

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where $f^{\pm}(t)$, $g_{\pm}(t)$ *are the functions of Hölder class* H *given on* L *and* Λ, *respectively.*

Upper or lower indices "$\pm$" indicate the boundary values on contours of the cuts L (or Λ) from above (or from the left) and from below (or from the right), respectively.

A solution of the problem under consideration will be presented in two modes.

Mode I

In the boundary condition (1) we introduce $-\overline{t}$ instead of t. Taking into account that if t runs through the points of L in one direction (from the left to the right), then $-\overline{t}$ runs, because of the symmetry of the cuts, through the same points in the opposite direction (from the right to the left), and we obtain the conditions again on L:

$$\big[\varphi(-\overline{t})+\overline{\psi}(-t)\big]^{\pm}=f^{\pm}(-\overline{t}),\quad t\in L, \tag{1'}$$

Reasoning analogously, in the condition (2) we replace t by $\overline{t}$. Assuming that $t\in\Lambda$, we have

$$\big[\varphi(\overline{t})+\psi(t)\big]_{\pm}=g_{\pm}(\overline{t}),\quad t\in L, \tag{2'}$$

Summarizing and subtracting the boundary conditions first (1) and (1′) and then (2) and (2′), we obtain

$$\big[\varphi(t)+\overline{\psi}(-t)+\varphi(-\overline{t})+\overline{\psi(t)}\big]^{\pm}=f^{\pm}(t)+f^{\pm}(-\overline{t}),\ \ t\in L, \tag{3}$$

$$\big[\varphi(t)-\overline{\psi}(-t)-\varphi(-\overline{t})+\overline{\psi(t)}\big]^{\pm}=f^{\pm}(t)-f^{\pm}(-\overline{t}),\ \ t\in L, \tag{4}$$

$$\big[\varphi(t)+\overline{\psi}(t)+\varphi(\overline{t})+\overline{\psi(t)}\big]_{\pm}=g_{\pm}(t)+g_{\pm}(\overline{t}),\ \ t\in\Lambda, \tag{5}$$

$$\big[\varphi(t)-\overline{\psi}(t)-\varphi(\overline{t})+\overline{\psi(t)}\big]_{\pm}=g_{\pm}(t)-g_{\pm}(\overline{t}),\ \ t\in\Lambda. \tag{6}$$

The above conditions can be written in the form

$$\begin{gathered}
\Omega_1^{+}(t)-\Omega_1^{-}(t)=f_1(t),\quad \Omega_2^{+}(t)+\Omega_2^{-}(t)=f_2(t),\quad t\in L,\\
\Omega_{1+}(t)-\Omega_{1-}(t)=g_1(t),\quad \Omega_{2+}(t)-\Omega_{2-}(t)=g_2(t),\quad t\in\Lambda,\\
\Omega_3^{+}(t)-\Omega_3^{-}(t)=f_3(t),\quad \Omega_4^{+}(t)+\Omega_4^{-}(t)=f_4(t),\quad t\in L,\\
\Omega_{3+}(t)+\Omega_{3-}(t)=g_3(t),\quad \Omega_{4+}(t)+\Omega_{4-}(t)=g_4(t),\quad t\in\Lambda,
\end{gathered}$$

or, what is the same thing,

$$\Omega_{2\nu-n}^{+}(t)-(2n-1)\Omega_{2\nu-n}^{-}(t)=f_{2\nu-n}(t),\quad t\in L, \tag{7}$$

$$\big[\Omega_{2\nu-n}(t)\big]_{+}+(2\nu-3)\big[\Omega_{2\nu-n}(t)\big]_{-}=g_{2\nu-n}(t),\quad t\in\Lambda, \tag{8}$$

$$n=0,1,\quad \nu=1,2.$$

where we have introduced the notation

$$\Omega_{2\nu-n}(z) = \varphi(z) + (2\nu-3)\overline{\psi}(-z) - (2n-1)\big[\overline{\psi}(z) + (2\nu-3)\varphi(-z)\big], \quad (9)$$

$$f_{2\nu-n}(t) = f^{+}(t) + (2\nu-3)f^{+}(-\overline{t}) - (2n-1)\big[f^{-}(t) + (2\nu-3)f^{-}(-\overline{t})\big],$$
$$g_{2\nu-n}(t) = g_{+}(t) + (2\nu-3)g_{-}(t) - (2n-1)\big[g_{+}(\overline{t}) + (2\nu-3)g_{-}(\overline{t})\big],$$
$$n=0,1, \qquad \nu=1,2.$$

The functions $\Omega_{2\nu-n}(z)$, $n=0,1$, $\nu=1,2$, are, because of the symmetry of the cuts, piecewise holomorphic with boundary lines $L\cup\Lambda$ in the domain S; they must satisfy supplementary conditions

$$\Omega_{2\nu-n}(z) = -(2n-1)(2\nu-3)\Omega_{2\nu-n}(-z), \quad n=0,1, \quad \nu=1,2. \qquad (10)$$

As is seen, the solution of the problem (1), (2) is reduced to four mixed boundary value problems of linear conjugation. By means of canonical functions [1], we can reduce each of the problems to the simplest problem, i.e., to the problem of finding a piecewise holomorphic function with a prescribed jump.

We introduce into consideration canonical functions $T_{2\nu-2}(z)$, $n=0,1$, $\nu=1,2$, which by themselves are solutions of the homogeneous problem [1]

$$T^{+}_{2\nu-n}(t) - (2n-1)T^{-}_{2\nu-n}(t) = 0, \quad t\in L,$$
$$\big[T_{2\nu-n}(t)\big]_{+} + (2n-3)\big[T_{2\nu-n}(t)\big]_{-} = 0, \quad t\in\Lambda,$$
$$n=0,1, \quad \nu=1,2.$$

A particular solution of that problem we represent as follows:

$$T_{2\nu-n}(z) = \big[\chi(z)\big]^{1-n}\big[Q(z)\big]^{\nu-1}, n=0,1 \quad \nu=1,2, \qquad (11)$$

where

$$\chi(z)\equiv\chi_0(z) = \prod_{k=1}^{p}\big(z^2-a_k^2\big)^{-1/2}\big(z^2-b_k^2\big)^{-1/2},$$

and

$$Q(z)\equiv Q_0(z) = \prod_{j=1}^{m}\big(z^2+\alpha_j^2\big)^{-1/2}\big(z^2+\beta_j^2\big)^{-1/2}$$

are the solutions corresponding to the unbounded ones (having integrable singularities) in the vicinity of all ends.

If we are required to construct solutions, bounded in the vicinity of the given ends $c_1, c_2, \dots, c_q$, $0\le q\le 2p$, $d_1, d_2, \dots, d_s$, $0\le s\le m$ and taking into account that in the vicinity of symmetric ends a solution is sought in one and the same class, then

$$\chi(z) \equiv \chi_q(z) = \prod_{k=1}^{q} \left(z^2 - c_k^2\right)^{1/2} \prod_{k=q+1}^{2p} \left(z^2 - c_k^2\right)^{-1/2}, \tag{12}$$

$$Q(z) \equiv Q_s(z) = \prod_{j=1}^{s} \left(z^2 + d_j^2\right)^{1/2} \prod_{j=s+1}^{2m} \left(z^2 + d_j^2\right)^{-1/2}; \tag{13}$$

here c_k and d_j denote respectively the ends of the cuts L and Λ which are enumerated arbitrarily.

The functions $\chi_q(z)$ and $Q_s(z)$ are holomorphic in the domain S if under these functions we mean certain branches; for example, those for which

$$\lim_{z\to\infty} \left[z^{2(p-q)}\chi_q(z)\right] = 1, \quad \lim_{z\to\infty} \left[z^{2(m-s)}Q_s(z)\right] = 1.$$

Notice here that the relations

$$\chi_q(z) = \overline{\chi}_q(z) = \chi_q(-z), \quad Q_s(z) = \overline{Q}_s(z) \tag{14}$$

hold.

The solutions $T_{2\nu-n}(z)$ expressed by the functions $\chi_0(z)$, $Q_0(z)$ or $\chi_q(z)$, $Q_s(z)$, are canonical functions of the classes h_0, and h_{q+s}, respectively ([1], §78).

We seek for solutions of the problem (7),(8) belonging to the class h_{q+s} and vanishing at infinity.

Using canonical functions, the boundary conditions (7),(8) take the form

$$\left[\frac{\Omega_{2\nu-n}(t)}{T_{2\nu-n}(t)}\right]^+ - \left[\frac{\Omega_{2\nu-n}(t)}{T_{2\nu-n}(t)}\right]^- = \frac{f_{2\nu-n}(t)}{T^+_{2\nu-n}(t)}, \quad t \in L, \tag{15}$$

$$\left[\frac{\Omega_{2\nu-n}(t)}{T_{2\nu-n}(t)}\right]_+ - \left[\frac{\Omega_{2\nu-n}(t)}{T_{2\nu-n}(t)}\right]_- = \frac{g_{2\nu-n}(t)}{[T_{2\nu-n}(t)]_+}, \quad t \in \Lambda, \tag{16}$$

$$n = 0, 1, \quad \nu = 1, 2.$$

Consequently, we have obtained the boundary value problem of finding a piecewise holomorphic function $\Omega_{2\nu-n}(z)/T_{2\nu-n}(z)$ by means of the given jump.

Having found the functions $\Omega_{2\nu-n}(z)$, $n = 0, 1$, $\nu = 1, 2$, by virtue of (9), we find main unknown functions by the formulas

$$\varphi(z) = \frac{1}{4}\sum_{n=0}^{1}\sum_{\nu=1}^{2} \Omega_{2\nu-n}(z), \quad \psi(z) = \frac{1}{4}\sum_{n=0}^{1}\sum_{\nu=1}^{2} (1-2n)\overline{\Omega}_{2\nu-n}(z). \tag{17}$$

A solution of the problem (15),(16) we represent as follows ([1], [2] § 110):

$$\Omega_{2\nu-n}(z) = \Omega^0_{2\nu-n}(z) + \Omega^*_{2\nu-n}(z), \quad n = 1, 0, \quad \nu = 1, 2, \tag{18}$$

where $\Omega^0_{2\nu-n}(z)$ is a general solution of the homogeneous problem ($f_{2\nu-n}(t) = g_{2\nu-n}(t) = 0$), and $\Omega^*_{2\nu-n}(z)$ is a particular solution of the inhomogeneous problem (15),(16). The functions $\Omega^0_{2\nu-n}(z)$ and $\Omega^*_{2\nu-n}(z)$ must, obviously, satisfy the relations (supplementary conditions) (10).

A general solution of the given class of the homogeneous problem (15), (16) which may have the pole at infinity, is given by the formula ([1], [2] § 110)

$$\Omega^0_{2\nu-n}(z) = T_{2\nu-n}(z)P_{2\nu-n}(z) = \big[\chi(z)\big]^{1-n}\big[Q(z)\big]^{\nu-1}P_{2\nu-n}(z), \qquad (19)$$
$$n = 1, 0, \quad \nu = 1, 2$$

where $P_{2\nu-n}$ are arbitrary polynomials. In order to obtain a solution vanishing at infinity, taking into account the behaviour $T_{2\nu-n}(z)$ and also (10),(14), we choose polynomials $P_{2\nu-n}(z)$ as follows:

$$P_1(z) = 0 \quad \text{for any values} \quad q \quad \text{and} \quad s,$$
$$P_2(z) = \sum_{j=0}^{q_0} M_{2j+1}z^{2j+1}, \quad \text{for} \quad q < p, \quad q_0 = p - q - 1, \qquad (20)$$

if $q \geq p$, then $P_2(z) = 0$;

$$P_3(z) = \sum_{j=0}^{m_0} M^0_{2j+1}z^{2j+1}, \quad \text{for} \quad s < m, \quad m_0 = m - s - 1, \qquad (21)$$

if $s \geq m$, then $P_3(z) = 0$;

$$P_4(z) = \sum_{j=0}^{n_0} M_{2j}z^{2j}, \quad \text{for} \quad q + s < p + m, \quad n_0 = p + m - q - s - 1, \qquad (22)$$

if $q + s \geq p + m$, then $P_4(z) = 0$.

M_{2j+1}, M_{2j}, M^0_{2j+1} are arbitrary constants.

A particular solution of the inhomogeneous problem (15),(16) is defined by the formula ([1], §31)

$$\frac{\Omega^*_{2\nu-n}}{T_{2\nu-n}(z)} = \frac{1}{2\pi i}\int_L \frac{f_{2\nu-n}(\tau)}{T^+_{2\nu-n}(\tau)}\frac{d\tau}{\tau - z} + \frac{1}{2\pi i}\int_\Lambda \frac{g_{2\nu-n}(\tau)}{[T_{2\nu-n}(\tau)]_+}\frac{d\tau}{\tau - z} =$$
$$= \frac{1}{2\pi i}\int_L \frac{f^+(\tau) - (2n-1)f^-(\tau)}{T^+_{2\nu-n}(\tau)}\Big[\frac{1}{\tau - z} - \frac{(2n-1)(2\nu-3)}{\tau + z}\Big]\,d\tau +$$
$$+\frac{1}{2\pi i}\int_\Lambda \frac{g_+(\tau) + (2\nu-3)g_-(\tau)}{[T_{2\nu-n}(\tau)]_+}\Big[\frac{1}{\tau - z} - \frac{(2n-1)(2\nu-3)}{\tau + z}\Big]\,d\tau, \qquad (23)$$
$$n = 0, 1, \quad \nu = 1, 2.$$

For $n = 1$, $\nu = 1$ the solution always exists, $\Omega_1(z)$ is almost bounded ([1], §77) in the vicinity of all ends; moreover, it will be bounded in the vicinity of those ends at which $f^+(t)$, $g_+(t)$ and $f^-(t)$, $g_-(t)$ take the same values.

For $q \leq p$, $s \leq m$, $q+s \leq p+m$ the solution (23) of the class h_{q+s}, vanishing at infinity, always exists.

In the remaining cases the following conditions must be satisfied: for $n=0$, $\nu=1$, $q>p$

$$\int_L \frac{f^+(\tau)+f^-(\tau)}{\chi^+(\tau)}\tau^{2\mu}\,d\tau + \int_\Lambda \frac{g_+(\tau)-g_-(\tau)}{\chi(\tau)}\tau^{2\mu}\,d\tau = 0, \tag{24}$$
$$\mu = 0,1,\dots,q-p-1;$$

for $n=0$, $\nu=2$, $q+s>p+m$

$$\int_L \frac{f^+(\tau)+f^-(\tau)}{\chi^+(\tau)Q(\tau)}\tau^{2\mu+1}\,d\tau + \int_\Lambda \frac{g_+(\tau)+g_-(\tau)}{\chi(\tau)Q_+(\tau)}\tau^{2\mu+1}\,d\tau = 0, \tag{25}$$
$$\mu = 0,1\dots,q+s-p-m-1;$$

for $n=1$, $\nu=2$, $s>m$

$$\int_L \frac{f^+(\tau)-f^-(\tau)}{Q(\tau)}\tau^{2\mu}\,d\tau + \int_\Lambda \frac{g_+(\tau)+g_-(\tau)}{Q_+(\tau)}\tau^{2\mu}\,d\tau = 0, \tag{26}$$
$$\mu = 0,1\dots,s-m-1.$$

It can be easily seen that the supplementary conditions (10) are satisfied.

Further, according (17) and (18), a general solution of the initial problem can be represented in the form

$$\varphi(z) = \frac{1}{4}\sum_{n=0}^{1}\sum_{\nu=1}^{2}\Big\{\Omega^*_{2\nu-n}(z) + \big[\chi_q(z)\big]^{1-n}\big[Q_s(z)\big]^{\nu-1}P_{2\nu-n}(z)\Big\}, \tag{27}$$

$$\psi(z) = \\ = \frac{1}{4}\sum_{n=0}^{1}\sum_{\nu=1}^{2}(1-2n)\Big\{\overline{\Omega}^*_{2\nu-n}(z) + \big[\chi_q(z)\big]^{1-n}\big[Q_s(z)\big]^{\nu-1}\overline{P}_{2\nu-n}(z)\Big\}, \tag{28}$$

where $\Omega^*_{2\nu-n}(z)$ and $P_{2\nu-n}(z)$ are given by formulas (20), (21), (22) and (23).

Thus as regards a solution of the initial problem of the class h_{q+s}, vanishing at infinity and representable by formulas (27) and (28), we can conclude the following:

When $q+s<p+m$, for $q \leq p$, $s<m$ or $q<p$, $s \leq m$ a solution is always exists; for $q>p$, $s<m$ or $q<p$, $s>m$ for a solution to exist, the conditions (24) or (26) must, respectively, be satisfied.

When $q+s=p+m$, for $q=p$, $s=m$ a unique solution always exists; for $q>p$, $s<m$ or $q<p$, $s>m$ a solution exists only under the conditions (24) or (26) respectively.

When $q+s>p+m$, for $q>p$, $s>m$ the solution exists if the conditions (24), (25) and (26) are satisfied; for $q>p$, $s \leq m$ or $q \leq p$, $s>m$

the solution exists if the conditions (24), (25) or (25), (26) are satisfied, respectively.

Mode II

On the other hand, the boundary conditions can be written in the form

$$\varphi^+(t)+\overline{\psi}^{\,-}(t)=f^+(t),\;\; \varphi^-(t)+\overline{\psi}^{\,+}(t)=f^-(t),\;\; t\in L,$$
$$\varphi_+(t)+\overline{\psi}_-(-t)=g_+(t),\;\; \varphi_-(t)+\overline{\psi}_+(-t)=g_-(t),\;\; t\in\Lambda.$$

Summarizing and subtracting the above equalities, we obtain

$$\phi_0^+(t)+\phi_0^-(t)=f^+(t)+f^-(t),\;\; \Psi_0^+(t)-\Phi_0^-(t)=f^+(t)-f^-(t),\;\; t\in L, \tag{29}$$
$$\phi_+(t)+\phi_-(t)=g_+(t)+g_-(t),\;\; \Psi_+(t)-\Psi_-(t)=g_+(t)-g_-(t),\;\; t\in\Lambda, \tag{30}$$

where we have introduced the notation

$$\phi_0(z)=\varphi(z)+\overline{\psi}(z),\;\; \Psi_0(z)=\varphi(z)-\overline{\psi}(z),$$
$$\phi(z)=\varphi(z)+\overline{\psi}(-z),\;\; \Psi(z)=\varphi(z)-\overline{\psi}(-z).$$

Representing the boundary conditions (3), (4), (5) and (6) in the same notation, we obtain

$$\big[\phi(t)+\phi(-\overline{t})\big]^{\pm}=f^{\pm}(t)+f^{\pm}(-\overline{t}),\;\; \big[\Psi(t)-\Psi(-\overline{t})\big]^{\pm}= \\ =f^{\pm}(t)-f^{\pm}(-\overline{t}),\;\; t\in L,$$

$$\big[\phi_0(t)+\phi_0(-\overline{t})\big]_{\pm}=g_{\pm}(t)+g_{\pm}(-\overline{t}),\;\; \big[\Psi_0(t)-\Psi_0(-\overline{t})\big]_{\pm}= \\ =g_{\pm}(t)-g_{\pm}(-\overline{t}),\;\; t\in\Lambda.$$

Grouping the above-obtained conditions with the conditions (29) and (30), we can conclude that finding of the functions $\phi_0(z)$, $\Psi_0(z)$, $\phi(z)$, $\Psi(z)$ is reduced to four problems with the combined boundary conditions [3], when the conditions of the problem of linear conjugation are given on one contour, and the conditions with a shift are prescribed on the other contour. Indeed, when t runs over the borders of the cuts L (or Λ) in one direction (from the left to the right), then $-\overline{t}$ (or $\overline{t}$) runs over the same cuts L (or Λ) in the opposite direction. Let

$$\alpha_-(t)=-\overline{t},\;\; \text{for}\;\; t\in L \;\;\text{and}\;\; \alpha_-(t)=\overline{t},\;\; \text{for}\;\; t\in\Lambda$$

be an inverse Carleman's shift [4]. Then the boundary conditions can be rewritten as follows:

$$\Big[\Psi\big[\alpha_-(t)\big] - \Psi(t)\Big]^{\pm} = f^{\pm}(-\bar{t}) - f^{\pm}(t), \quad t \in L,$$
$$\Psi_+(t) - \Psi_-(t) = g_+(t) - g_-(t), \quad t \in \Lambda,$$
$$\Big[\Psi_0\big[\alpha_-(t)\big] - \Psi_0(t)\Big]_{\pm} = g_{\pm}(\bar{t}) - g_{\pm}(t), \quad t \in \Lambda,$$
$$\Psi_0^+(t) - \Psi_0^-(t) = f^+(t) - f^-(t), \quad t \in L.$$

Assuming

$$\frac{\phi_0(z)}{\chi(z)} \equiv \phi_0^*(z), \quad \frac{\phi(z)}{Q(z)} \equiv)\phi^*(z),$$

where canonical functions are given by formulas (12) and (13), and taking into account the relations (14), for the functions $\phi_0^*(z)$ and $\phi^*(z)$ we obtain analogous combined conditions.

These problems, having an independent interest, are easily solvable. Towards this end, it is sufficient to make use of reasoning of Mode I. Clearly, our final results coincide, as it was expected, with the results obtained in Mode I.

Consequently, the solution of the initial problem in Mode II is treated as that of the combined Carleman's problem and of linear conjugation.

Finally, we note that the case for $\psi(z) \equiv \varphi(z)$ has been considered in [5].

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