Proceedings of A.Razmadze
Mathematical Institute,
Vol. **123**, 2000

ON SOME PROPERTIES OF SEMI-COMPACT AND S-CLOSED BITOPOLOGICAL SPACES

I. DOCHVIRI

Abstract. The notions of semi-compact and S-closed topological spaces for the bitopological case are generalized. The characterizing properties of $(i,j)-S$ closed bitopological spaces in connection with $(i,j)-$semi-compact, (i,j)-quasi-H-compact and p-extremally disconnected spaces, where $i,j\in\{1,2\}$, $i\neq j$ are given.

J. Kelly's work [1] is considered fundamental in the development of the theory of bitopological spaces. The author specifies the (X,τ_1,τ_2) bitopological space as a natural generalization of the topological space-a set X possessing two independent topologies τ_1 and τ_2.

Let $\mathcal{J}$ represent some topological property. Then by $(i,j)-\mathcal{J}$ we denote the analogue of property $\mathcal{J}$ for the bitopological space (X,τ_1,τ_2) (throughout this paper it will be assumed that $i,j\in\{1,2\}$, $i\neq j$). By $p-\mathcal{J}$ we denote the conjunction $(1,2)-\mathcal{J}\wedge(2,1)-\mathcal{J}$, i.e., an "absolute" bitopological analogue of the property $\mathcal{J}$.

In the present work we study properties of (i,j)-semi-open sets in connection with (i,j)-irresolute and (i,j)-weakly-semi-continuous maps and investigate some properties of (i,j)-semi-compact and $(i,j)-S$-closed bitopological spaces connected naturally with the notion of an (i,j)-semi-open covering.

Defintion 1. A subset A of the bitopological space (X,τ_1,τ_2) is said to be (i,j)-semi-open, if there exists an open set $O\in\tau_i$ such that $O\subset A\subset$

1991 *Mathematics Subject Classification.* 54E55.
Key words and phrases. Bitopological space, semi-compact bitopological space, $(i,j)-$ S-closed bitopological space, p-extremally disconnected bitopological space.

$\tau_j - clO$ [2], where $\tau_j - clO$ is the closure of the set O with respect to the topology τ_j.

A class of all (i,j)-semi-open sets in the bitopological space (X, τ_1, τ_2) we denote by $(i,j) - SO(X)$.

From Definition 1 it follows that $A \in (i,j) - SO(X)$ if and only if $A \subset \tau_j - cl\tau_i - \operatorname{int} A$, where $\tau_i - \operatorname{int} A$ is the interior of the set A with respect to the topology τ_i.

The set K is the (i,j)-semi-neighbourhood of the point $x \in X$ in the bitopological space (X, τ_1, τ_2) if there exists the set $M \in (i,j) - SO(X)$ such that $x \in M \subset K$. This implies that $A \in (i,j) - SO(X)$ if and only if A is the (i,j)-semi-neighbourhood of every its point.

Defintion 2. A complement to the set $A \in (i,j) - SO(X)$ is called (i,j)-semi-closed in (X, τ_1, τ_2) and we write $X \backslash A \in (i,j) - SC(X)$.

The set K is (i,j)-semi-closed if and only if there exists a τ_i-closed set F in (X, τ_1, τ_2) such that the inclusion $\tau_j - \operatorname{int} F \subset K \subset F$ is fulfilled.

In [2] it was emphasized that if $\{A_\alpha\}_{\alpha \in \wedge}$ is a family of (i,j)-semi-open sets, then $\underset{\alpha \in \Lambda}{\cup} A_\alpha \in (i,j) - SO(X)$, but if $A_1,\ A_2 \in (i,j) - SO(X)$ then $A_1 \cap A_2 \notin (i,j) - SO(X)$, generally speaking. It is easy to see that the intersection of (i,j)-semi-closed sets is (i,j)-semi-closed, but a finite union of (i,j)-semi-closed sets is not (i,j)-semi-closed, in general.

Defintion 3. A point $q \in X$ in the bitopological space (X, τ_1, τ_2) is said to be an (i,j)-semi-adherent point for the subset $A \subset X$ if every its (i,j)-semi-open neighbourhood intersects with the set A.

By $(i,j) - \operatorname{Sint} A$ $((i,j) - SclA)$ we denote a set of all (i,j)-semi-interior $((i,j)$-semi-adherent) points of the set $A \subset X$.

It is known that

$$A \in (i,j) - SO(X) \Leftrightarrow A = (i,j) - \operatorname{Sint} A;$$
$$A \in (i,j) - SC(X) \Leftrightarrow A = (i,j) - SclA.$$

Theorem 1. *For every subset A in the bitopological space (X, τ_1, τ_2) we have* [3]*:*

$$(i,j) - \operatorname{Sint} A = A \cap \tau_j - cl\tau_i - \operatorname{int} A;$$
$$(i,j) - SclA = A \cup \tau_j - \operatorname{int} \tau_i - clA.$$

Proof. First of all we note that the set $A \cap \tau_j - cl\tau_i - \operatorname{int} A$ is (i,j)-semi-open. Indeed, $\tau_i - \operatorname{int} A \subset A \cap \tau_j - cl\tau_i - \operatorname{int} A \subset \tau_j - cl\tau_i - \operatorname{int} A$. Take any point $x \in A \cap \tau_j - cl\tau_i - \operatorname{int} A \subset A$. Then the set $A \cap \tau_j - cl\tau_i - \operatorname{int} A$ is its (i,j)-semi-open neighbourhood, and $A \cap \tau_j - cl\tau_i - \operatorname{int} A \subset A$. Hence $x \in (i,j) - \operatorname{Sint} A$.

Conversely, let $x \in (i,j) - \operatorname{Sint} A$. Then there exists a set $U \in (i,j) - SO(X)$ such that $x \in U \subset A$. From $U \in (i,j) - SO(X)$ follows $U \subset$

$\tau_j - cl\tau_i - \operatorname{int} U$ and therefore $x \in A \cap \tau_j - cl\tau_i - \operatorname{int} U \subset A \cap \tau_j - cl\tau_i - \operatorname{int} A$. The proof of the equality $(i,j) - SclA = A \cup \tau_j - \operatorname{int} \tau_i - clA$ is obtained from the easily verifiable equation $(i,j) - SclA = X \backslash (i,j) - \operatorname{Sint}(X \backslash A)$. Consider the set $(i,j) - SclA = X \backslash (i,j) - \operatorname{Sint}(X \backslash A) = X \backslash (X \backslash A) \cap \tau_j - cl\tau_i - \operatorname{int}(X \backslash A) = A \cup [X \backslash \tau_j - cl\tau_i - \operatorname{int}(X \backslash A)] = A \cup X \backslash (\tau_j - cl[X \backslash \tau_i - clA]) = A \cup \tau_j - \operatorname{int}[X \backslash \tau_i - cl(X \backslash A)] = A \cup \tau_j - \operatorname{int} \tau_i - clA$. $\square$

Let there be given bitopological spaces (X, τ_1, τ_2) and (Y, γ_1, γ_2). The mapping $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is said to be pairwise continuous (or p-continuous) if the mappings $f : (X, \tau_i) \to (Y, \gamma_i)$ $i = 1, 2$ are continuous [4]. However, if the mappings $f : (X, \tau_i) \to (Y, \gamma_i)$ $i = 1, 2$ are open simultaneously, then $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is called p-open.

Proposition 1. *If the mapping $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is p-continuous and p-open, then*

$$f^{-1}((i,j) - SclA) \subset (i,j) - Sclf^{-1}(A)$$

for every subset A from the bitopological space (Y, γ_1, γ_2).

Proof. Consider the preimage of the set $(i,j) - SclA$, where $A \subset Y$ is any subset. Then $f^{-1}[(i,j) - SclA] = f^{-1}(A \cup \gamma_j - \operatorname{int} \gamma_i - clA) = f^{-1}(A) \cup \tau_j - \operatorname{int} f^{-1}(\gamma_i - clA)$, since the mapping f is p-continuous. We now take advantage of Sikorski's theorem [5]: if the mapping $f : (X, \tau) \to (Y, \gamma)$ is open, then $f^{-1}(\gamma - clA) \subset \tau - clf^{-1}(A)$. From the latter it is clear that $f^{-1}[(i,j) - SclA] \subset f^{-1}(A) \cup \tau_j - \operatorname{int} \tau_i - clf^{-1}(A) = (i,j) - Sclf^{-1}(A)$. $\square$

Defintion 4. We say that the bitopological space (X, τ_1, τ_2) is of type $(i,j) - sT_0$ if for any two different points $x \neq y$ from X there exists $U \in (i,j) - SO(X)$ such that $x \in U$ and $y \notin U$.

Defintion 5. The mapping $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is said to be (i,j)-irresolute if for every $U \in (i,j) - SO(Y)$ the sets set $f^{-1}(U) \in (i,j) - SO(X)$ [2].

Proposition 2. *If the mapping $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is an (i,j)-irresdute injection, and the bitopological space (Y, γ_1, γ_2) is of type $(i,j) - sT_0$, then (X, τ_1, τ_2) is of type $(i,j) - sT_0$ as well.*

Proof. Let $x \neq y$ be any two different points from X. Then $f(x) \neq f(y)$. By the condition, there exists $U \in (i,j) - SO(Y)$ such that $f(x) \in U$ and $f(y) \notin U$. Hence the set $f^{-1}(U) \in (i,j) - SO(X)$ contains a point x and $y \notin f^{-1}(U)$. $\square$

Defintion 6. We say that the bitopological space (X, τ_1, τ_2) is of type $(i,j) - sT_2$ if for any two different points $x \neq y$ from X there exist $U; W \in (i,j) - SO(X)$ such that $x \in U$, $y \in W$ and $U \cap W = \varnothing$.

In this work we will write $(X, \tau_1 R \tau_2)$ if topologies in the space (X, τ_1, τ_2) are connected by the relation $\tau_1 - cl\tau_1 - \operatorname{int} U = \tau_2 - cl\tau_1 - \operatorname{int} U$ for any subset $U \subset X$.

Defintion 7. The mapping $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is said to be (i, j)-weakly-semi-continuous, if for any point x_0 and every set $V \in \gamma_i$ containing the point $f(x_0)$ there exists a neighbourhood $U \in (i, j) - SO(X)$ of the point x_0 such that $f(U) \subset \gamma_j - clV$ [6].

Theorem 2. *If the mapping* $f : (X, \tau_1 C \tau_2) \to (Y, \gamma_1 R \gamma_2)$ *is* $(1, 2)$*-weakly-semi-continuous* [6] *injection and* (Y, γ_1) *is the Urysohn space, then the bitopological space* (X, τ_1, τ_2) *belongs to the type* $(1, 2) - sT_2$.

Proof. Suppose that $x \neq y$ are any two points in (X, τ_1, τ_2). Since f is injective, $f(x) \neq f(y)$. By the condition, (Y, γ_1) is the rysohn space, hence there exist the sets $U, W \in \gamma_1$ such that $f(x) \in U$, $f(y) \in W$ and $\gamma_1 - clU \cap \gamma_1 - clW = \varnothing$. From the fact that $\gamma_1 - cl\gamma_1 - \operatorname{int} U = \gamma_2 - cl\gamma_1 - \operatorname{int} U$, we have $\gamma_1 - clU \cap \gamma_1 - clW = \gamma_2 - clU \cap \gamma_2 - clW = \varnothing$. Using Definition 7, it is not difficult to see that there exist $(1, 2)$-semi-open sets $x \in M$, $y \in N$ such that $f(M) \subset \gamma_2 - clU$, $f(N) \subset \gamma_2 - clV$. Thus $f(M) \cap f(N) = \varnothing$, and consequently, $M \cap N = \varnothing$. $\square$

If topologies in the bitopological space (X, τ_1, τ_2) are connected by the relation $\tau_1 - clU \subset \tau_2 - clU$ for any $U \in \tau_1$, then the use will be made of the notation $(X, \tau_1 C \tau_2)$ [7].

Theorem 3. *If the mapping* $f : (X, \tau_1 C \tau_2) \to (Y, \gamma_1, \gamma_2)$ *is* (1)*-continuous and* (2)*-closed, then it is* $(1, 2)$*-weakly-semi-continuous as well.*

Proof. Take any point $x_0 \in X$ and any $V \in \gamma_1$-open neighbourhood containing the point $f(x_0) \in Y$. Since f is (1)-continuous, there exists $x_0 \in U$ τ_1-open set such that $f(U) \subset V$. Taking into account that in the $(X, \tau_1 C \tau_2)$ bitopological space $U \subset \tau_1 - clU \subset \tau_2 - clU$, we have $\tau_1 - clU \in (1, 2) - SO(X)$. Thus there exists the set $x_0 \in W$, where $W \equiv \tau_1 - clU$. Using the (2)-closeness of the mapping f, we obtain $f(W) \subset \gamma_2 - clV$. $\square$

Proposition 3. *If the mapping* $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ *is* (i, j)*-irresolute and* $g : (Y, \gamma_1, \gamma_2) \to (Z, \omega_1, \omega_2)$ *is* (i, j)*-weakly-semi-continuous, then the composition* $g \circ f : (X, \tau_1, \tau_2) \to (Z, \omega_1, \omega_2)$ *is* (i, j)*-weakly-semi- continuous as well.*

Proof. Let $x_0 \in X$ be any point and $A \subset Z$ be an ω_i-open subset such that $z_0 = g \circ f(x_0) \in A$. Since the mapping g is (i, j)-weakly-semi- continuous, there exists the (i, j)-semi-neighbourhood $U \subset Y$ of the point $y_0 = f(x_0)$ such that $g(U) \subset \omega_j - clA$. Since the mapping f is (i, j) -irresolute, then $f^{-1}(U) \in (i, j) - SO(X)$ and $x_0 \in f^{-1}(U)$. Consequently we have $g \circ f[f^{-1}(U)] = g(U) \subset \omega_j - clA$. $\square$

Defintion 8. A bitopological space (X, τ_1, τ_2) is said to be (i, j)-semi-compact if its every (i, j)-semi-open $\{U_\alpha\}_{\alpha\in\Lambda}$ cover admits finite subcover.

Theorem 4. *Every (i, j)-semi-compact bitopological space (X, τ_1, τ_2) is a τ_i-compact topological space.*

Proof. Suppose $\{U_\alpha\}_{\alpha\in\Lambda}$ is a τ_i-open covering of (X, τ_1, τ_2). Then it automatically is an (i, j)-semi-open covering. Since the bitopological space (X, τ_1, τ_2) is (i, j)-semi-compact, we can extract from it a finite subcovering. □

Defintion 9. The bitopological space (X, τ_1, τ_2) is said to be (i, j)-quasi-H- closed ($(i, j) - QHC$), if for any $\{U_\alpha\}_{\alpha\in\Lambda}$ open covering of the bitopological space (X, τ_1, τ_2) there exists a finite subfamily $\{U_{\alpha k}\}_{k=1,\dots,n}$ of the covering $\{U_\alpha\}_{\alpha\in\Lambda}$, such that $X = \overset{n}{\underset{k=1}{\cup}} \tau_j - clU_{\alpha k}$ [8].

Proposition 4. *If the bitopological space (X, τ_1, τ_2) is (i, j)-semi-compact, then it represents the $(i, j) - QHC$ space.*

Proof. By Theorem 4 the bitopological space (X, τ_1, τ_2) is τ_i-compact, hence if $\{U_\alpha\}_{\alpha\in\Lambda}$ is τ_i-open covering of (X, τ_1, τ_2), then there exists a finite subcover $\{U_{\alpha k}\}_{k=1,\dots,n} \subset \{U_\alpha\}_{\alpha\in\Lambda}$, i.e., $X = \overset{n}{\underset{k=1}{\cup}} U_{\alpha k} \subset \overset{n}{\underset{k=1}{\cup}} \tau_j - clU_{\alpha k}$. Thus (X, τ_1, τ_2) represents the $(i, j) - QHC$ space. □

Theorem 5. *The bitopological space (X, τ_1, τ_2) is said to be (i, j)-semi-compact if and only if every its centered family of (i, j)-semi-closed sets has a non-empty intersection.*

Proof. To prove the necessity we suppose to the contrary that the bitopological space (X, τ_1, τ_2) is (i, j)-semi-compact, and the family of (i, j)-semi-closed sets $\{F_\alpha\}_{\alpha\in\Lambda}$ is such that $\cap_{\alpha\in\Lambda} F_\alpha = \varnothing$. Then the family $\{U_\alpha\}_{\alpha\in\Lambda}$ consisting of sets $U_\alpha = X \backslash F_\alpha$ represents the (i, j)-semi-open covering of the space (X, τ_1, τ_2). By Definition 8, we can extract from the covering $\{U_\alpha\}_{\alpha\in\Lambda}$ a finite subcovering $\{U_{\alpha k}\}_{k=1,\dots,n} \subset \{U_\alpha\}_{\alpha\in\Lambda}$. Hence $X = \overset{n}{\underset{k=1}{\cup}} U_{\alpha k} = \overset{n}{\underset{k=1}{\cup}} [X \backslash F_{\alpha k}] = X \backslash [\overset{n}{\underset{k=1}{\cap}} F_{\alpha k}]$, i.e., $\overset{n}{\underset{k=1}{\cap}} F_{\alpha k} = \varnothing$ and this contradicts the $\{F_\alpha\}_{\alpha\in\Lambda}$ family's property of being centered.

Let us prove the necessity. Let $\{U_\alpha\}_{\alpha\in\Lambda}$ be an (i, j)-semi-open covering of the botopological space (X, τ_1, τ_2). Then the family $\{F_\alpha = X \backslash U_\alpha\}_{\alpha\in\Lambda}$ consists of (i, j)-semi-closed sets, such that $\cap_{\alpha\in\Lambda} F_\alpha = \varnothing$. Then there exists a finite subfamily $\{F_{\alpha k}\}_{k=1,\dots,n}$ of the family $\{F_\alpha\}_{\alpha\in\Lambda}$, such that $\overset{n}{\underset{k=1}{\cap}} F_{\alpha k} = \varnothing$. Hence there exists a finite subcovering $\{U_\alpha = X \backslash F_{\alpha k}\}_{k=1,\dots,n}$ of the covering $\{U_\alpha\}_{\alpha\in\Lambda}$. □

Proposition 5. *If the mapping $f : (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is (i, j)-irresolute and maps the (i, j)-semi-compact bitopological space (X, τ_1, τ_2) onto (Y, γ_1, γ_2), then the (Y, γ_1, γ_2) bitopological space (Y, γ_1, γ_2) is (i, j)-semi-compact.*

Proof. If $\{U_\alpha\}_{\alpha\in\Lambda}$ is the (i,j)-semi-open covering of the bitopological space (Y,γ_1,γ_2), i.e., $Y=\cup_{\alpha\in\Lambda}U_\alpha$, then the family $\{f^{-1}(U_\alpha)\}_{\alpha\in\Lambda}$ covers the bitopological space (X,τ_1,τ_2) and is the family of (i,j)-semi-open sets. Since the bitopological space (X,τ_1,τ_2) is (i,j)-semi-compact, then from $\{f^{-1}(U_\alpha)\}_{\alpha\in\Lambda}$ we can extract a finite subcovering. From the latter, taking into account the surjectivity of the mapping f, we obtain that the bitopological space (Y,γ_1,γ_2) is (i,j)-semi-compact. □

In work [9] T. Thompson generalized the notion of H-closed space by semi-open sets. The obtained in such a way analogues of H-closed spaces are called S-closed topological spaces. In the same paper the author outlined interesting connections between the S-closed and compact spaces.

We consider the $(i,j)-S$-closed bitopological spaces represent in some sense the generalization of the S-closed spaces and give some charaterizing properties of $(i,j)-S$-closed bitopological spaces.

Defintion 10. The bitopological space (X,τ_1,τ_2) is said to be $(i,j)-S$-closed, if every its (i,j)-semi-open covering $\{U_\alpha\}_{\alpha\in\Lambda}$ contains a finite family $\{U_{\alpha k}\}_{k=1,\dots,n}$ such that $X=\bigcup\limits_{k=1}^{n}\tau_j-clU_{\alpha k}$.

In particular, if $\tau_i=\tau_j$, then Definition 10 coincides with the definition of the S-closed topological space given in [9].

We can easily see from Definitions 9 and 10 that the following proposition below is valid.

Proposition 6. *If the bitopological space (X,τ_1,τ_2) is $(i,j)-S$-closed, then it is $(i,j)-QHC$ space.*

Note that the inverse statement is, in general, invalid.

It is not difficult to prove that there takes place

Proposition 7. *The bitopological space (X,τ_1,τ_2) is $(i,j)-S$ closed if and only if for every family $\{F_\alpha\}_{\alpha\in\Lambda}$ of (i,j)-semi-closed sets satisfying the condition $\cap_{\alpha\in\Lambda}F_\alpha=\varnothing$ there exists a finite subfamily $\{F_{\alpha k}\}_{k=1,\dots,n}\subset\{F_\alpha\}_{\alpha\in\Lambda}$ such that $\bigcap\limits_{k=1}^{n}\tau_j-\operatorname{int}F_{\alpha k}=\varnothing$.*

Proposition 8. *Every (i,j)-semi-compact bitopological space is $(i,j)-S$ closed.*

The proof follows immediately from Definitions 8 and 10.

A set of the bitopological space is said to be (i,j)- regularly open if $A=\tau_i-\operatorname{int}\tau_j-clA$. Complement of the (i,j)-regularly open set to the whole space is called (i,j)-regularly closed.

Proposition 9. *If the bitopological space (X,τ_1,τ_2) is $(i,j)-S$ closed, then any (j,i)-regularly closed covering contains a finite subcovering.*

Proof. Let the family $\{U_\alpha\}_{\alpha\in\Lambda}$ be the (j,i)-regularly closed covering of the bitopological space (X,τ_1,τ_2) (i.e., $U_\alpha=\tau_j-cl\tau_i-\operatorname{int}U_\alpha$ for every $\alpha\in\Lambda$). Then $U_\alpha\in(i,j)-SO(X)$ for $\alpha\in\Lambda$, and therefore the family $\{U_\alpha\}_{\alpha\in\Lambda}$ is the (i,j)-semi-open covering. By virtue of the $(i,j)-S$-closedness of the bitopological space (X,τ_1,τ_2) there exists a subfamily $\{U_{\alpha k}\}_{k=1,\dots,n}$ of the family $\{U_\alpha\}_{\alpha\in\Lambda}$ such that $X=\bigcup\limits_{k=1}^{n}\tau_j-clU_{\alpha k}=\bigcup\limits_{k=1}^{n}U_{\alpha k}$. □

Corollary 1. *The bitopological space $(X,\tau_1R\tau_2)$ is $(1,2)-S$-closed if and only if every τ_1-regularly closed covering contains a finite subcovering.*

Proof. The necessity follows from Proposition 9.

Let $\{W_\alpha|W_\alpha\in(1,2)-SO(X)\}_{\alpha\in\Lambda}$ be the covering of the space $(X,\tau_1R\tau_2)$. Then the family $W'=\{\tau_2-clW_\alpha=\tau_2-cl\tau_1-\operatorname{int}W_\alpha\}_{\alpha\in\Lambda}$ covers the bitopological space $(X,\tau_1R\tau_2)$. Since $\tau_1-cl\tau_1-\operatorname{int}W_\alpha=\tau_2-cl\tau_1-\operatorname{int}W_\alpha$, the family W' is a τ_1-regularly closed covering. Hence we can extract from it a canonical subcovering $\{\tau_2-clW_{\alpha k}\}_{k=1,\dots,n}\subset W'$ i.e., $X=\bigcup\limits_{k=1}^{n}\tau_2-clW_{\alpha k}$. □

Defintion 11. The bitopological space (X,τ_1,τ_2) is said to be p-extremely disconnected if for every $O\in\tau_i$ the sets $\tau_j-clO\in\tau_i$ [7].

Theorem 6. *If the bitopological space (X,τ_1,τ_2) is p-extremely disconnected and τ_i-compact, then it is $(i,j)-S$-closed.*

Proof. Let the family $\{U_\alpha\}_{\alpha\in\Lambda}$ be an (i,j)-semi-open covering of the bitopological space (X,τ_1,τ_2). Then the family $\{W_\alpha\}_{\alpha\in\Lambda}$, where $W_\alpha=\tau_j-cl\tau_i-\operatorname{int}U_\alpha$ for every $\alpha\in\Lambda$, is the covering of the bitopological space (X,τ_1,τ_2). Taking into account the fact that the space (X,τ_1,τ_2) is p-extremally disconnected, the family $\{W_\alpha\}_{\alpha\in\Lambda}$ represents an τ_i open covering. Using the τ_i-compactness of (X,τ_1,τ_2), we obtain the existence of a finite subfamily $\{W_{\alpha k}\}_{k=1,\dots,n}\subset\{W_\alpha\}_{\alpha\in\Lambda}$, such that $X=\bigcup\limits_{k=1}^{n}W_{\alpha k}$. Since $W_\alpha=\tau_j-clU_\alpha=\tau_j-cl\tau_i-\operatorname{int}U_\alpha$ for every $\alpha\in\Lambda$, we have $X=\bigcup\limits_{k=1}^{n}W_{\alpha k}=\bigcup\limits_{k=1}^{n}\tau_j-clU_{\alpha k}$. □

Theorem 7. *If the mapping $f:(X,\tau_1,\tau_2)\to(Y,\gamma_1,\gamma_2)$ is (i,j)-irresolute and maps j-continuously the $(i,j)-S$-closed bitopological space (X,τ_1,τ_2) onto (Y,γ_1,γ_2), then the space (Y,γ_1,γ_2) is $(i,j)-S$-closed.*

Proof. Assume the family $\{U_\alpha\}_{\alpha\in\Lambda}$ is an (i,j)-semi-open covering of the space (Y,γ_1,γ_2). Then the family $\{f^{-1}(U_\alpha)\}_{\alpha\in\Lambda}$ covers the bitopological space (X,τ_1,τ_2), where $f^{-1}(U_\alpha)\in(i,j)-SO(X)$ for every $\alpha\in\Lambda$.

Because of $(i,j)-S$-closedness of the bitopological space (X,τ_1,τ_2), the covering $\{f^{-1}(U_\alpha)\}_{\alpha\in\Lambda}$ contains a finite subfamily $\{f^{-1}(U_\alpha)\}_{k=1,\dots,n}$ such that $X=\bigcup\limits_{k=1}^{n}\tau_j-clf^{-1}(U_{\alpha k})$. Then $Y=f[\bigcup\limits_{k=1}^{n}\tau_j-clf^{-1}(U_{\alpha k})]=\bigcup\limits_{k=1}^{n}f[\tau_j-$

$clf^{-1}(U_{\alpha k})]$ and since the mapping f is j-continuous

$$Y = \bigcup_{k=1}^{n} f[\tau_j - clf^{-1}(U_{\alpha k})] =$$
$$= \bigcup_{k=1}^{n} \gamma_j - clf[f^{-1}(U_{\alpha k})] = \bigcup_{k=1}^{n} \gamma_j - clU_{\alpha k}.$$

Thus the bitopological space (Y, γ_1, γ_2) is $(i, j) - S$-closed. □

References

1. J. Kelly, Bitopological spaces. *Proc. London Math. Soc.* **13**(1963), No. 3, 71–89.

2. S. N. Maheshwari and R. Prasad, On pairwise irresolute functions. *Mathematica* **18**(41), (1976), No. 2, 169–172.

3. I. Dochviri, On semi-compact and pre-compact bitopological spaces. *Bull. Georgian Acad. Sci.* **161**(2000), No. 3, 403–406.

4. J. Swart, Total disconnectedness in bitopological spaces and product bitopological spaces. *Indag. Math.* **33**(1971), 135–145.

5. R. Sikorski, Closed homomorphisms and interior mappings. *Fund. Math.* **41**(1955), 12–20.

6. V. Popa and T. Noiri, Characterizations of weakly quasicontinuous functions in bitopological spaces. *Mathematica* **39**(62), (1997), No. 2, 293–297.

7. B. Dvalishvili, Investigations of bitopologies and their applications. *Doctoral Dissertation*, *Tbilisi*, 1994.

8. M. N. Mukherjee and S. Raychaudhuri, Some applications of θ-closure operators. *Indian J. Pure Appl. Math.* **26**(5)(1995), 433–439.

9. T. Thompson, S-closed spaces. *Proc. of Amer. Math. Soc.* **60**(1976), 335–338.

(Received 15.01.1999, revised 22.06.2000)

Author's address:
Faculty of Mechanics and Mathematics,
I. Javakhishvili Tbilisi State University,
2, University St.,
Tbilisi 380043
Georgia