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ON THE SUMMABILITY OF FOURIER TRIGONOMETRIC SERIES OF VARIABLE ORDER

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Let

$$\frac{a_0}{2}+\sum_{k=1}^{\infty} a_k \cos kx + b_k \sin kx \tag{1}$$

be the Fourier series of a summable function $f(x)$, and the triangular matrix

$$\Lambda = \|\lambda_n(k)\|,$$

such that $\lambda_n(0)=1$ and $\lambda_n(n+p)=0$ for $n \geq 0$ and $p \geq 1$ be given.

Consider the means of the series (1):

$$t_n(x;f,\Lambda)=\frac{a_0}{2}+\sum_{k=1}^{n}\lambda_n(k)(a_k\cos kx+b_k\sin kx).$$

If at a point x there exists the limit

$$\lim_{n\to\infty} t_n(x;f,\Lambda)=S,$$

then we say that the series (1) is Λ-summable at the point X to S.

Let $\{\alpha_n\}$ be a sequence of numbers from the interval $[0,1]$, and for $0 \leq k \leq n$ the numbers $\lambda_n(k)$ be defined either by the equality

$$\lambda_n(k)=\frac{A_{n-k}^{\alpha_n}}{A_n^{\alpha_n}}, \quad \text{where} \quad A_n^{\alpha}=\frac{(\alpha+1)(\alpha+2)\cdots(\alpha+n)}{n!}, \tag{2}$$

or by

$$\lambda_n(k)=\Big(1-\frac{k}{n+1}\Big)^{\alpha_n}. \tag{3}$$

If $\alpha_n=\alpha$ for any n, and the relations (2) are fulfilled, then it is clear that the Λ-summability is, in fact, the Cesaro summbility (C,α), but if are fulfilled the relations (3), then the Λ-summability is the Riesz summability of order α.

In [1] we formulated some theorems for the Λ-sumability in case $\alpha_n \to 0+$ as $n\to\infty$ (see [1], Theorems 3 and 4), from which, in particular, it follows that in the both cases (2) and (3) the following theorem is valid.

Theorem A. *For any sequence of numbers $\alpha_n \to 0+$, as $n\to\infty$, there exists a continuous function $f(x)$, such that the sequence of means $\{t_n(x;f,\Lambda)\}$ diverges at the point x_0.*

In connection with the above Theorem, V. Temlyakov put the question regarding the possibility of constructing, for every continuous $f(x)$, a sequence of numbers $\{\alpha_n\}$ with $\alpha_n \to 0+$, such that the sequence of means $\{t_n(x;f,\Lambda)\}$ is convergent to $f(x)$ at every point. The answer is positive.

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For the case (2), as well as for the case (3), the following theorem is valid.

Theorem. *For any continuous function $f(x)$, there exists a sequence of numbers $\alpha_n \downarrow 0$ as $n \to \infty$, such that the equality*

$$\lim_{n\to\infty} t_n(x; f, \Lambda) = f(x)$$

is fulfilled at every point x.

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References

1. Sh. Tetunashvili, On the iterated summability of trigonometric Fourier series. *Proc. A. Razmadze Math. Inst.* **139**(2005), 142–144.

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