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ON EQUILIBRIUMS IN THE LIQUID FLOW BETWEEN TWO PERMEABLE CYLINDERS IN THE PRESENCE OF A TRANSVERSE PRESSURE GRADIENT

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The stability of flow of a viscous incompressible liquid between two rotating permeable cylinders is considered when the flow is under the action of a constant pressure gradient round the cylinders. As is known [1], the Navier-Stokes and continuity equations in the cylindrical coordinates (r, θ, z) have exact solution with the velocity vector $\vec{v}_0 = \{v_{0r}, v_{0\theta}, v_{0z}\}$ and pressure p_0:

$$v_{0r} = \frac{\varkappa_0}{r}, \quad v_{0z} = 0,$$

$$v_{0\theta} = \begin{cases} \dfrac{1}{2\varkappa}\left(\dfrac{\partial p_0}{\partial \theta}\right)\left(-r + A_1 r^{\varkappa+1} + \dfrac{B_1}{r}\right) + A r^{\varkappa+1} + \dfrac{B}{r}, & \varkappa \neq -2, \\[2mm] \dfrac{1}{4}\left(\dfrac{\partial p_0}{\partial \theta}\right)\left(r - \dfrac{A_1' \ln r + 1}{r}\right) + \dfrac{A' \ln r + 1}{r}, & \varkappa = -2, \end{cases}$$

$$\frac{1}{\mathrm{Re}}\frac{\partial p_0}{\partial r} = \frac{v_{0\theta}^2}{r} + \frac{\varkappa_0^2}{r^3},$$

$$A_1 = \frac{R^2 - 1}{R^{\varkappa+2} - 1}, \quad B_1 = \frac{R^2(R^\varkappa - 1)}{R^{\varkappa+2} - 1}, \quad A = \frac{\Omega R^2 - 1}{R^{\varkappa+2} - 1}, \tag{1}$$

$$B = \frac{R^2(R^\varkappa - \Omega)}{R^{\varkappa+2} - 1}, \quad A_1' = \frac{R^2 - 1}{\ln R}, \quad A' = \frac{\Omega R^2 - 1}{\ln R}, \quad R = \frac{R_2}{R_1}, \; \Omega = \frac{\Omega_2}{\Omega_1},$$

$$\varkappa_0 = \frac{s}{\Omega_1 R_1^2}, \quad s = R_i v_{0r}|_{r=R_i} \; (i=1,2), \quad \mathrm{Re} = \frac{\Omega_1 R_1^2}{\nu}, \quad \frac{\partial p_0}{\partial \theta} = P\theta = \mathrm{const},$$

$\varkappa = s/\nu$ is the radial Reynolds number, ν-kinematic viscosity.

In [1], the neutral curves are constructed which show that after the basic stationary flow (1) losses its stability depending on the parameters $\varkappa$, Ω, $P\theta$, α and m of the problem (α and m are, respectively, the axial and azimuthal wave numbers), for $R = 2$ there may take place both the rotationally-symmetric motions in the form of vortices and the running azimuthal waves with periods 2π or π.

Our aim is to investigate the regimes appearing in a small neighborhood of the point of intersection of neutral curves which correspond to these two types of instability.

The main object of our analysis of stability and bifurcation is a motor subsystem of amplitude equations ([2,3]) which is a generalization of the well-known Landau's amplitude equation.

Using this method, it becomes possible to investigate complex regimes in the Couette flow, in the flows between heated cylinders and between permeable cylinders ([2–7]).

As our calculations show, unlike the Couette flow and that between permeable cylinders, depending on the parameters of the problem, the neutral curves may intersect not

only when the cylinders rotate in different directions, but when they rotate in one and the same direction. Such intersection one can observe for weakly rotating cylinders under positive values of a pressure gradient and also when the liquid is injected through the porous outer cylinder, for example, for $\varkappa = -1$, $P\theta = 6$, $\Omega = 0,0401$, $\alpha = 3$, Re $= 185, 2375$. As is known, at those points one can expect the rise of possible complex regimes.

It is stated that under certain values of parameters of the problem, for the basic flow (1) can be realized all possible cases of equilibrium occurring on the invariant planes, as well as those of general state ([2, 3]). A number of calculations of equilibrium are performed both for $\varkappa > 0$ (liquid injection through the inner cylinder) and for $\varkappa < 0$ (liquid injection through the outer cylinder) for the positive and negative values of pressure gradient. Under certain values of parameters of the problem to these equilibriums there correspond vertices, purely azimuthal waves, spiral waves, a pair of mixed azimuthal waves. Their stability and bifurcation are investigated. Moreover, we managed to find that some equilibriums may have branches limiting cycles, i.e. isolated solutions of the motor subsystem. To every such solution there corresponds triple-frequency quasi-periodic motion regime.

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