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ON THE STABILITY OF LONG ORTHOTROPIC SHELLS OF ROTATION, CLOSE BY THEIR FORM TO CYLINDRICAL ONES

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A more precise system of equations of stability for long orthotropic shells of rotation, close by their form to cylindrical ones, is obtained. In an isotropic case, when a midsurface generatrix of the shell is a parabolic function, the obtained equation differs from the well-known ([1]) by additive terms which may be of the same order as another terms.

The shells with midsurface formed by the rotation of some sufficiently smooth curve around the z-axis of the rectangular system of coordinates x, y, z and origin at the middle of the segment of the axis of rotation, are considered. Note that the radius of midsurface cross-section of the shell is defined by the equality $R = r + \delta_0 F(\xi)$, where $\xi = z/r$; $F(\xi)$ is the positive function given on the interval $(-l/r; l/r)$, such that $F(\pm l/r) = 0$; $\max F(\xi) = 1$, $|F'(\xi)| \lesssim 1$; $L = 2l$ is the length of the shell, r is the radius of the edge cross-section, δ_0 is a small parameter characterizing maximal deviation from the cylinder. For $\delta_0 > 0$, the midsurface generatrix is convex, while for $\delta_0 < 0$, it is concave. It is assumed that

$$(\delta_0/r)^2, (\delta_0/l)^2 \ll 1. \tag{1.1}$$

The midsurface equation has parametrically the form $x = R(\xi)\cos\varphi$, $y = R(\xi)\sin\varphi$, $z = r\xi$, where φ is the angular coordinate. This implies that the coefficients of the first quadratic midsurface form are $A^2 = r^2 + \delta_0 (F')^2$ and $B^2 = R(\xi)^2$. On the basis of our assumptions, the second term in the expression for A^2 can be neglected. Consequently, $A \approx r$, $B = R(\xi)$. The principle radii of curvature have the forms

$$k_1 = 1/R_1 = -R''/r^2, \quad k_2 = 1/R_2 = 1/R(\xi). \tag{1.2}$$

It is assumed that the shell is under the action of normal load which is distributed uniformly over the whole lateral surface of the shell and of meridional stresses distributed over the edge sections of the shell. The stressed state induced by that load is called basic state. The stability of that state is investigated.

In deducing the equations of stability of long shells we proceed from the nonlinear equations of equilibrium with regard for midsurface deformations ([2]). Further linearization together with the use of improved relations of elasticity for orthotropic shells ([3]), as well as taking into account the fact that for the forms of stability loss there takes place the relation

$$\frac{\partial^2 f}{\partial \xi^2} \ll \frac{\partial^2 f}{\partial \varphi^2} \quad (f = u, v, w), \tag{1.3}$$

where u, v and w are, respectively, the axial, angular and radial components of displacement, characterizing the form of stability loss, we obtain the following system of

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equations of stability of orthotropic shells:

$$\frac{\partial^4 u}{\partial \varphi^4}=\Big[\Big(-\frac{R''}{r}\Big)\frac{E_1}{E_2}+\nu_1\Big]\frac{\partial^3 w}{\partial \xi^3}+\Big[\Big(\frac{E_1}{G}-\nu_1\Big)\Big(-\frac{R''}{r}\Big)-1\Big]\frac{R}{r}\frac{\partial^3 w}{\partial\xi\,\partial\varphi^2}, \tag{1.4}$$

$$\frac{\partial^4 v}{\partial \varphi^4}=\Big[1+\nu_1\Big(-\frac{R''}{r}\Big)\Big]\frac{\partial^3 w}{\partial \varphi^3}+\Big[\Big(\frac{E_1}{G}-\nu_1\Big)-\Big(-\frac{R''}{r}\Big)\frac{E_1}{E_2}\Big]\frac{\partial^3 w}{\partial\xi^2\,\partial\varphi}, \tag{1.5}$$

$$\begin{aligned}&\varepsilon\Big(\frac{\partial^8 w}{\partial\varphi^8}+2\frac{\partial^6 w}{\partial\varphi^6}+\frac{\partial^4 w}{\partial\varphi^4}\Big)+\frac{E_1}{E_2}\Big\{\frac{\partial^4 w}{\partial\xi^4}+\frac{\partial^2}{\partial\xi^2}\Big[\Big(-\frac{R''}{r}\Big)\frac{\partial^2 w}{\partial\varphi^2}\Big]-\\ &\quad-\frac{R''}{r}\frac{\partial^4 w}{\partial\xi^2\varphi^2}+\Big(\frac{R''}{r}\Big)^2\frac{\partial^4 w}{\partial\varphi^4}\Big\}+\frac{T_1^0}{E_2h}\Big(\frac{\partial^6 w}{\partial\xi^2\,\varphi^4}-\frac{\partial^4 w}{\partial\xi^2\,\varphi^2}\Big)+\\ &\quad+\frac{T_2^0}{E_2h}\Big(\frac{\partial^6 w}{\partial\varphi^6}+\frac{\partial^4 w}{\partial\varphi^4}\Big)+\frac{S^0}{E_2h}\Big(\frac{\partial^6 w}{\partial\xi\,\partial\varphi^5}+\frac{\partial^4 w}{\partial\xi\,\varphi^3}\Big)=0,\end{aligned} \tag{1.6}$$

where E_1, E_2, ν_1, ν_2 are the elasticity moduli and Poisson coefficients in the axial and angular directions ($E_1\nu_2 = E_2\nu_1$), G are displacement moduli, T_1^0, T_2^0, S^0 are the normal and shear stresses of the basic state.

For the shells whose midsurface generatrix is defined by the parabolic function

$$F(\xi) = 1-\xi^2(r/l)^2,$$

the resolving equation (1.6) takes the form

$$\begin{aligned}&\varepsilon\Big(\frac{\partial^8 w}{\partial\varphi^8}+2\frac{\partial^6 w}{\partial\varphi^6}+\frac{\partial^4 w}{\partial\varphi^4}\Big)+\frac{E_1}{E_2}\Big(\frac{\partial^4 w}{\partial\xi^4}+4\delta\frac{\partial^4 w}{\partial\xi^2\,\partial\varphi^2}+4\delta^2\frac{\partial^4 w}{\partial\varphi^4}\Big)+t_1^0\Big(\frac{\partial^6 w}{\partial\xi^2\,\partial\varphi^4}-\frac{\partial^4 w}{\partial\xi^2\,\partial\varphi^2}\Big)+\\ &\quad+t_2^0\Big(\frac{\partial^6 w}{\partial\varphi^6}+\frac{\partial^4 w}{\partial\varphi^4}\Big)+2s^0\Big(\frac{\partial^6 w}{\partial\xi\,\partial\varphi^5}+\frac{\partial^4 w}{\partial\xi\,\partial\varphi^3}\Big)=0,\\ &\varepsilon=h^2/12r^2(1-\nu_1\nu_2),\quad \delta=\delta_0 r/l^2,\quad t_i^0=T_i^0/E_2h\ (i=1,2),\quad s^0=S^0/E_2h.\end{aligned}$$

The additive terms in the above equation as compared to the well-known equation for isotropic shell ([1]), are the sixth term which, owing to the equation (1.2), are of the same order as the fifth term, and moreover, the additive are also the eighth and the last terms.

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