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THE PROBLEM OF PLATE BENDING FOR THE REGION WITH A PARTIALLY UNKNOWN BOUNDARY

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The problem of bending of a plate with a partially unknown boundary for a weakened by an unknown hole square whose neighborhoods at the vertices are cut out by unknown equal smooth arcs, is considered.

To the linear segments of the boundary are attached rigid strips. The plate bends by the concentrated moments applied to midpoints of the strips. An unknown part of the boundary is free from external actions.

The problem is formulated as follows: Find unknown parts of the boundary under the condition that the normal tangential moment on them takes constant value.

Since the problem is axisymmetric, the solution of the problem reduces by the symmetry principle to the problem of plate bending for a curvilinear hexagon $A_1A_2A_3A_4A_5A_6$ whose two sides A_2A_3 and A_5A_6 are the unknown arcs, and the sides A_1A_2, A_3A_4, A_4A_5 and A_6A_1 are the linear segments of the boundary.

As is known [1,2], the investigation of the problem of plate bending is reduced to that of finding analytic functions $\varphi(z)$ and $\psi(z)$ in the region S, which is, in fact, a middle plane of the plate with certain boundary conditions which in our case have the form

$$\begin{aligned}&\operatorname{Re}\big[e^{-i\alpha(t)}(\varphi(t)+t\,\overline{\varphi'(t)}+\overline{\psi(t)}\big]=d,\quad t\in\Gamma,\\&\operatorname{Re}\big[e^{-i\alpha(t)}(m\varphi(t)+t\,\overline{\varphi'(t)}+\overline{\psi(t)}\big]=c(t),\quad t\in\Gamma,\end{aligned}\tag{1}$$

$$m\varphi(t)+t\,\overline{\varphi'(t)}+\overline{\psi(t)}=B(t),\quad t\in\breve{A_2A_3}\cup\breve{A_5A_6},\tag{2}$$

$$\operatorname{Re}\,\varphi'(t)=\frac{M_\theta}{4D(1-\nu)}\,,\tag{3}$$

where $c(t)$ and $B(t)$ are the given piecewise constant functions, $d=$const is an unknown constant, $\alpha(t)$ is the size of the angle made by the outer normal n and the Ox-axis, m,ν,D are elastic constants, Γ is the union of known segments of the boundary, and A_2A_3 and A_5A_6 are the unknown arcs.

It is proved that

$$\varphi(z)=kz.\tag{4}$$

Taking into account equality (4) and performing certain transformations of the boundary conditions (1) and (2), we obtain

$$\begin{aligned}&\operatorname{Re}\Big[\frac{k}{2}\,e^{-\frac{\pi}{4}i}t+e^{\frac{\pi}{4}i}\psi(t)\Big]=\begin{cases}0, & t\in A_1A_2\cup\breve{A_2A_3},\\-\dfrac{p}{2}\,, & t\in A_4A_5\cup\breve{A_5A_6},\end{cases}\\&\operatorname{Im}\Big[\frac{k}{2}\,e^{-\frac{\pi}{4}i}t+e^{\frac{\pi}{4}i}\psi(t)\Big]=\begin{cases}ka, & t\in A_3A_4,\\\dfrac{p}{2}\,, & t\in\breve{A_5A_6},\end{cases}\end{aligned}\tag{5}$$

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$$
\begin{aligned}
&\mathrm{Re}\Big[\frac{k}{2}\,e^{-\frac{\pi}{4}i}t-\psi(t)e^{\frac{\pi}{4}i}\Big]=\begin{cases} ka, & t\in A_1A_2,\\ \dfrac{p}{2}\,, & t\in A_4A_5,\end{cases}\\
&\mathrm{Im}\Big[\frac{k}{2}\,e^{-\frac{\pi}{4}i}t-\psi(t)e^{\frac{\pi}{4}i}\Big]=\begin{cases} 0, & t\in A_3A_4\cup \overset{\frown}{A_2A_3},\\ -\dfrac{p}{2}\,, & t\in \overset{\frown}{A_5A_6}\cup A_6A_1,\end{cases}
\end{aligned}
\tag{6}
$$

where $2a$ is the side length of the square, p is the given constant, and k is the unknown constant.

Let the function $z=\omega(\zeta)$ map the half-plane $\mathrm{Im}\,\zeta>0$ onto the region S. If we introduce the notation

$$
\Phi(\zeta)=\frac{k}{2}\,e^{-\frac{\pi}{4}i}\omega(\zeta)+e^{\frac{\pi}{4}i}\psi[\omega(\zeta)],\ \ \mathrm{Im}\,\zeta>0, \tag{7}
$$

$$
\Psi(\zeta)=\frac{k}{2}\,e^{-\frac{\pi}{4}i}\omega(\zeta)-e^{\frac{\pi}{4}i}\psi[\omega(\zeta)],\ \ \mathrm{Im}\,\zeta>0, \tag{8}
$$

then the boundary value problems (5) and (6) are reduced to the Keldysh-Sedov problems whose solutions are given by the formulas

$$
\begin{aligned}
\Phi(\zeta)&=\frac{X_1(\zeta)}{2\pi}\,i\Bigg[\int\limits_{-a_2}^{-a_1}\frac{2ak\,d\zeta}{|X_1(\xi)|(\xi-\zeta)}+\int\limits_{-a_1}^{a_1}\frac{p\,d\zeta}{|X_1(\xi)|(\xi-\zeta)}+c\Bigg],\quad \mathrm{Im}\,\zeta>0,\\
\Psi(\zeta)&=-\frac{X_2(\zeta)}{2\pi}\Bigg[\int\limits_{a_1}^{a_2}\frac{2ak\,d\zeta}{|X_2(\xi)|(\xi-\zeta)}+\int\limits_{-a_1}^{a_1}\frac{p\,d\zeta}{|X_2(\xi)|(\xi-\zeta)}-c\Bigg],\quad \mathrm{Im}\,\zeta>0,\\
&A_1=\omega(a_1),\quad A_2=\omega(a_2),
\end{aligned}
$$

where $X_1(\xi)$, $X_2(\xi)$ are the canonical functions.

It is proved that the problem has an infinite number of bounded solutions. From formulas (7) and (8) we find that

$$
\omega(\xi)=\frac{\Phi(\zeta)+\Psi(\zeta)}{k}\,.
$$

The graphs of unknown parts of the boundary are constructed.

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