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N. Shavlakadze

THE CONTACT PROBLEM FOR PIECEWISE HOMOGENEOUS ELASTIC PLANE REINFORCED BY A SEMI-INFINITE INCLUSION CUTTING THE INTERFACE AT RIGHT ANGLES

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A great deal of works are devoted to the investigation of static contact problems for different bodies reinforced by elastic attachments or inclusions such, as covers of small thickness (see e.g. [1–5]). The exact or approximate solutions of the posed problems are obtained and the behavior of unknown contact stresses at the contact ends is studied ([6–8]).

In the present work we consider a piecewise homogeneous elastic plate, reinforced by a semi-infinite inclusion under the action of tangential stress $\tau_k^{(0)}(x)$. The problem is to find contact tangential stresses $\tau_k(x)$ along the line of contact and to establish their behavior at singular points.

The problem is formulated mathematically as follows: let an elastic body S occupy a plane of a complex variable $z = x + iy$ which has along the line $L = (-\infty, 1]$ the elastic inclusion of elastic modulus E_0, of thickness h_0, the Poisson coefficient ν_0, consist of two half-planes $S_1 = \{z | \operatorname{Re} z > 0, z \overline{\in} \bar{l}_1 = [0, 1]\}$ and $S_2 = \{z | \operatorname{Re} z < 0, z \overline{\in} \bar{l}_2 = (-\infty, 0]\}$ sealed along the axis $x = 0$. The quantities and functions referring to S_k will be denoted by the index k $(k = 1, 2)$, while the boundary values of functions on the upper and lower edges of L are supplied by the signs $(+)$ and $(-)$, respectively.

On the interface $x = 0$ we have the following conditions of equilibrium and continuity:

$$\sigma_x^{(1)} = \sigma_x^{(2)}, \quad \tau_{xy}^{(1)} = \tau_{xy}^{(2)}, \quad \frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial y}, \quad \frac{\partial v_1}{\partial y} = \frac{\partial v_2}{\partial y} \tag{1}$$

On the segments l_k we have the following conditions:

$$\frac{du_k^{(0)}(x)}{dx} = \frac{1}{E_s} \int_{x_k}^{x} [\tau_k(t) - \tau_k^{(0)}(t)] dt, \quad x \in l_k, \quad k = 1, 2 \tag{2}$$

($u_k^{(0)}(x)$ is the horizontal displacement of the inclusion points, $x_1 = 0$, $x_2 = -\infty$), $E_s = \frac{E_0 h_0}{1 - \nu_0^2}$.

The equilibrium equations for separate parts of inclusion have the form

$$\int_{-\infty}^{0} [\tau_2(t) - \tau_2^{(0)}(t)] dt = p_0, \quad p_0 - \int_{0}^{1} [\tau_1(t) - \tau_1^{(0)}(t)] dt = p$$

where p_0 and p are the unknown axial stresses at the points $x = 0$ and $x = 1$, respectively.

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Using the Kolosov-Muskhelishvili's formulas ([9]), for the complex potentials $\varphi_k(z)$ and $\psi_k(z)$ $(k=1,2)$, we obtain the boundary conditions

$$\begin{aligned} &\varphi_k^+(x)-\varphi_k^-(x)=if_k(x),\\ &\qquad\qquad\qquad\qquad\qquad x\in l_k,\\ &\psi_k^+(x)-\psi_k^-(x)=-i(\varkappa_k f_k(x)+xf_k'(x)) \end{aligned} \qquad (3)$$

where $f_k(x)=\frac{1}{1+\varkappa_k}\int_{x_k}^{x}\tau_k(t)dt$, $\varkappa_k=\frac{3-\nu_k}{1+\nu_k}$.

Solutions of the boundary value problems (3) are represented as

$$\varphi_k(z)=\frac{1}{2\pi}\int\limits_{l_k}\frac{f_k(t)dt}{t-z}+W_k(z),\quad \psi_k(z)=\frac{-1}{2\pi}\int\limits_{l_k}\frac{[\varkappa_k f_k(t)+tf_k'(t)]dt}{t-z}+Q_k(z),$$

where $W_k(z)$ and $Q_k(z)$ are analytic in S_k functions which are defined from the condition (1) by the methods of the theory of analytic functions.

If we substitute the representation for complex potentials into the condition (2), then after some transformations we will obtain a system of singular integro-differential equations on the semi-axis $x>0$. Using the method of the Fourier integral transformation, the system is reduced to the boundary value problem of linear conjugation ([10]). The solution is represented explicitly. The behavior of contact stresses in the neighborhood of singular points $z=1$ and $z=0$ is studied.

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Author's address:

A. Razmadze Mathematical Institute,
Georgian Academy of Sciences,
1, M. Aleksidze St, 0193 Tbilisi
Georgia