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TWO-WEIGHTED ESTIMATES FOR FOURIER MULTIPLIERS

Let $L_w^{p,\theta}$ be a weighted Triebel-Lizorkin space. (For the definition, see [1], [2]).

Let ϕ be a measurable function defined on $\mathbb{R}^2$.

Our aim is to present conditions for the pair (v, w) of weights ensuring the boundedness of the operator

$$T_\phi f(x) = F^{-1}(\phi \widehat{f})(x)$$

from $L_w^{p,\theta}(\mathbb{R}^2)$ to $L_v^{p,\theta}(\mathbb{R}^2)$.

Here F^{-1} denotes the inverse Fourier transform. The Fourier transforms will be considered in the framework of the theory of S'-distributions.

We need the definitions of some classes of pairs of weights.

Definition 1. Let $1 < p < \infty$. A pair (v, w) of weight functions on $\mathbb{R}^2$ is said to be of the class Ω_p if v is even and increasing on $(0, \infty)$ in each variable uniformly to another one, $w(x, y) = w_1(x)w_2(y)$, where $w_i (i = 1, 2)$ are even and increasing on $(0, \infty)$, and the condition

$$\sup_{a,b>0} \Big(\int_a^\infty \int_b^\infty \frac{v(x,y)}{(xy)^p}\, dx\, dy \Big)^{1/p} \Big(\int_0^a \int_0^b w^{1-p'}(x,y)\, dx\, dy \Big)^{1/p'} < \infty, \ p' = \frac{p}{p-1},$$

is fulfilled.

Definition 2. Let $1 < p < \infty$. We say that a weight pair (v, w) defined on $\mathbb{R}^2$ belongs to G_p if v is even and decreasing on $(0, \infty)$ in each variable uniformly to another one, $w(x, y) = w_1(x)w_2(y)$, where one-dimensional weights $w_i (i = 1, 2)$ are even and decreasing on $(0, \infty)$, and the weight pair (v, w) satisfies the condition

$$\sup_{a,b>0} \Big(\int_0^a \int_0^b v(x,y)\, dx\, dy \Big)^{1/p} \Big(\int_a^\infty \int_b^\infty \frac{w^{1-p'}(x,y)}{(xy)^{p'}}\, dx\, dy \Big)^{1/p'} < \infty.$$

Theorem 1. *Let $1 < p, \theta < \infty$, $\{\mu_m\}_m$, $m = (m_1, m_2) \in \mathbb{Z}^2$, be a family of measures such that*

$$\int_{\mathbb{R}^2} |d\mu_m| \le c, \quad m \in \mathbb{Z}^2,$$

for some positive constant c. Suppose that the measure function $\phi(\lambda_1, \lambda_2)$ is representable as

$$\phi(\lambda_1, \lambda_2) = \int_{-\infty}^{\lambda_1} \int_{-\infty}^{\lambda_2} d\mu(t_1, t_2)$$

on every set

$$Q_m = \big\{ \lambda = (\lambda_1, \lambda_2) : 2^{m_j} < |\lambda_j| \le 2^{m_j+1},\ j = 1, 2;\ m_j = 0, \pm1, \dots \big\}.$$

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Then from the condition $(v,w) \in \Omega_p \cup G_p$ *it follows that the operator* T_ϕ *is bounded from* $L_w^{p,\theta}(\mathbb{R}^2)$ *to* $L_v^{p,\theta}(\mathbb{R}^2)$.

Theorem 2. *Let* $1 < p < \theta$, ϕ *be continuous outside the coordinate axes and have there continuous derivatives*

$$\frac{\partial^k \phi}{\partial \lambda_1^{k_1} \partial \lambda_2^{k_2}}, \quad 0 < k_1 + k_2 = k \leq 2, \quad k_j = 0, 1; \;\; j = 1, 2.$$

Moreover, assume that

$$\left| \lambda_1^{k_1} \lambda_1^{k_2} \frac{\partial^k \phi}{\partial \lambda_1^{k_1} \partial \lambda_2^{k_2}} \right| \leq M$$

and the condition $(v,w) \in \Omega_p \cup G_p$ *holds. Then the operator* T_ϕ *is bounded from* $L_w^{p,\theta}(\mathbb{R}^2)$ *to* $L_v^{p,\theta}(\mathbb{R}^2)$.

References

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