Proc. A. Razmadze Math. Inst. 126(2001), 117–118.

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On the Growth Order of Integral Means Functions from Orlicz Class $L\Phi(L)(\mathbf{R}^2)$

(Reported on 7.09.2001)

Let $I=\prod\limits_{k=1}^{n}(a_k,b_k)$ be an arbitrary n-dimensional interval and $f\in L(R^n)$. The behaviour of integral means $\frac{1}{|I|}\int\limits_I$ is of importance in the harmonic analysis. Investigation of integral mean growth order is a point of great nicety in the harmonic analysis. The first results in this direction have been obtained by G. Karagulyan [1–2]. Later on this question was studied in the works of the author [3–4]. The present work gives estimates of growth order of integral mean functions from the Orlicz class $L\Phi(L)(R^2)$ which in the definite sense are final.

Let $E=[0,1]^2$. For two-dimentional interval $I=(a_1,b_1)\times(a_2,b_2)$ let $m(I)$ denote the lenght of its smallest side: $m(I)=\min\limits_{k=1,2}\{b_k-a_k\}$.

Theorem 1(A)**.** *For any function $f\in L(E)$*

$$\lim_{\substack{\delta(I)\to 0\\ x\in I}}\frac{1}{|I|\log(\frac{1}{m(I)})}\int\limits_I f=0.$$

Moreover, for any number $\lambda>0$

$$\Big|\Big\{x\in E:\sup_{\substack{\delta(I)<1\\ x\in I, I\subset E}}\frac{1}{|I|\log(\frac{2}{m(I)})}\int\limits_I|f|>\lambda\Big\}\Big|<c\int|f|,$$

where c is an absolute constant.

(B) ([1]) *Let $\varepsilon:[0,+\infty[\to[0,+\infty[$ and $\varepsilon(t)\to 0$, $t\to\infty$. There exists a nonnegative function $f\in L(E)$ such that*

$$\limsup_{\substack{\delta(I)\to 0\\ x\in I}}\frac{1}{|I|\varepsilon(\frac{1}{m(I)})\log(\frac{1}{m(I)})}\int\limits_I f=+\infty,\quad \text{a.e.}\quad x\in E.$$

Let $\Phi:[1,+\infty[\to[0,+\infty[$ be strictly increasing differentiable function such that $\Phi(e^t)$ is convex above $\Phi(t)=o(\log t)$ and $\Phi(t)\uparrow\infty$ for $t\uparrow\infty$. Suppose

$$\Phi^+(t)=\begin{cases}\Phi(t), & t>1,\\ 0, & 0\le t\le 1\end{cases}.$$

Theorem 2(A)**.** *For any function $f\in L\Phi(L)(E)$*

$$\lim_{\substack{\delta(I)\to 0\\ x\in I}}\frac{\Phi(\frac{1}{m(I)})}{|I|\log(\frac{1}{m(I)})}\int\limits_I f=0,\quad a.e.$$

2000 *Mathematics Subject Classification:* 42B25, 41A65.

Key words and phrases. Integral mean, the rate of growth to infinity, strong maximal function, Orlicz class.

Moreover, for any number $\lambda > 0$

$$\left|\left\{x \in E : \sup_{\substack{\delta(I)<1 \\ x\in I, I\subset E}} \frac{\Phi(\frac{1}{m(I)})}{|I|\log(\frac{2}{m(I)})} \int_I |f| > \lambda\right\}\right| < c \int \frac{|f|}{\lambda}\Big(1 + \Phi^+\Big(\frac{|f|}{\lambda}\Big)\Big),$$

where c *is a constant, depending only on* Φ.

(*B*) *Let* $\varepsilon : [0, +\infty[\to [0, +\infty[$ *and* $\varepsilon(t) \to 0$, $t \to \infty$. *There exists a nonnegative function* $f \in L\Phi(L)(E)$ *such that*

$$\limsup_{\substack{\delta(I)\to 0 \\ x\in I}} \frac{\Phi(\frac{1}{m(I)})}{|I|\varepsilon(\frac{1}{m(I)})\log(\frac{1}{m(I)})} \int_I f = +\infty, \quad \text{a.e.} \quad x \in E.$$

Corollary 1. *For rectangular partial sums* $S_{m,n}(f,x)$ *from a double trigonometric Fourier series of functions* $f \in L([-\pi,\pi]^2)$ *the relation*

$$S_{m,n}(f,x) = o\Big(\log m \log n \log(\max(m,n))\Big)$$

is fulfilled almost everywhere on $[-\pi,\pi]^2$ *for* $\min(m,n) \to \infty$.

Corollary 2. *For the Fejer sums* $q_{m,n}(f,x)$ *of a double trigonometric Fourier series of functions* $f \in L([-\pi,\pi]^2)$ *the relation*

$$S_{m,n}(f,x) = o\Big(\log(\max(m,n))\Big)$$

is fulfilled almost everywhere on $[-\pi,\pi]^2$ *for* $(m,n) \to \infty$.

Analogously we find that for $f \in L\Phi(L)([-\pi,\pi])$ *the relation*

$$S_{m,n}(f,x) = o\left(\frac{\log(\max(m,n)}{\Phi(\max(m,n)}\right)$$

is fulfilled almost everywhere on $[-\pi,\pi]^2$ *for* $\min(m,n) \to \infty$.

Similar estimates are valid for rectangular partial sums of double Fourier-Haar series as well.

Note (see [5]) that the estimates of Corollary 2 are final.

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