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The Boudedness of Discrete Operators in Weighted Spaces

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The optimal sufficient conditions for pairs of sequences guaranteing the validity of two-weight inequalities for a potential type discrete operator

$$(T_\gamma \beta)_n = \sum_{k\neq n} \frac{\beta_k}{|n-k|^{1-\gamma}} \ , \ \ 0<\gamma<1.$$

are presented.

The necessary and sufficient conditions for pairs of weights ensuring the validity of two-weight inequlities for the potentials and fractional maximal functions were investigated in [1]. Analogous problem for singular operators (in continous case) are studied in [2].

For two-weighted estimates for potentials defined on homogeneous groups see [3].

The presented results can be considered as a discrete analogous of well-known Sobolev's theorem [4].

Definition. Let $1<p<\infty$. A non-negative sequence $\{\rho_k\}$ belongs to the class $A_p(Z)$ if

$$\sup_{m,n\in Z} \frac{1}{(n-m+1)^p} \Big(\sum_{k=m}^{n} \rho_k\Big)\Big(\sum_{k=m}^{n} \rho_k^{1-p'}\Big)^{p-1} < \infty,$$

where $p'=\frac{p}{p-1}$ for every m and n $(m,n\in Z)$. Moreover, the sequence $\{\rho_k\}$ belongs to the class $A_1(Z)$ if

$$\frac{1}{n-m+1}\sum_{k=m}^{n} \rho_k \le c \min_{m\le k\le n} \rho_k,$$

for every m, n, $m\le n$.

The following theorems are valid:

Theorem 1. *Let $0<\gamma<1$, $1<p<\frac{1}{\gamma}$ and $\frac{1}{p}-\frac{1}{q}=\gamma$. Let $\{b_k\}$, $\{\sigma_k\}$, and $\{\rho_k\}$ be positive on Z^+ sequences, $\{\sigma_k\}$ is increasing, $\sigma_0=0$ and $\{\rho_k\}$ satisfying the condition $\{\rho_k\}\in A_{1+\frac{q}{p'}}(Z)$, $(p'=\frac{p}{p-1})$.*

If the following conditions are fulfilled:

1) *there exists a positive c_1 such that*

$$\sigma_{2k+1}^{p/q}\rho_k \le c_1 b_k$$

for every positive integer k;

2)

$$\sup_{n\ge 0}\Big(\sum_{k=n+1}^{\infty} \frac{\sigma_k\rho_k}{k^{(1-\gamma)q}}\Big)^{p/q}\Big(\sum_{k=0}^{n}\Big(\rho_k^{-\gamma p} b_k\Big)^{1-p'}\Big)^{p-1} < \infty,$$

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then there exists a positive constant c such that for every $\{\beta_k\}$ *we have*

$$\Big(\sum_{k=-\infty}^{+\infty}\big|(T_\gamma(\beta\rho^\gamma))_k\big|^q a_{|k|}\Big)^{1/q} \le c\Big(\sum_{k=-\infty}^{+\infty}|\beta_k|^p\, b_{|k|}\Big)^{1/p}, \tag{1}$$

where $a_k=\sigma_k\rho_k$, $b_k=u_k\rho_k$.

Theorem 2. *Let* $0<\gamma<1$, $1<p<\frac{1}{\gamma}$ *and* $\frac{1}{p}-\frac{1}{q}=\gamma$. *Assume that* $\{\sigma_k\}$ *and* $\{u_k\}$ *be positive, increasing on* Z^+ *sequences and* $\sigma_0=0$, $\{\rho_k\}$ *be a positive on* Z^+ *sequence with the condition* $\{\rho_{|k|}\}\in A_{1+\frac{q}{p'}}(Z)$, $a_k=\sigma_k\rho_k$, $b_k=u_k\rho_k$.

Then the inequality (1) *holds iff*

$$\sup_{n\ge 0}\Big(\sum_{k=n+1}^{\infty}\frac{\sigma_k}{k^{(1-\gamma)q}}\Big)^{p/q}\Big(\sum_{k=0}^{n}\big(\rho_k^{-\gamma p}b_k\big)^{1-p'}\Big)^{p-1}<\infty.$$

Theorem 3. *Let* $0<\gamma<1$, $1<p<\frac{1}{\gamma}$ *and* $\frac{1}{p}-\frac{1}{q}=\gamma$. *Let* $\{b_k t\}$, $\{\sigma_k\}$ *and* $\{\rho_k\}$ *be positive on* Z^+ *sequences,* $\{\sigma_k\}$ *is decreasing and* $a_k=\sigma_k\rho_k$. *Let* $\{\rho_k\}$ *be a positive on* Z^+ *sequence such that* $\{\rho_{|k|}\}\in A_{1+\frac{q}{p'}}(Z)$.

If the following conditions are fulfilled:

1) *there exists a positive* c_1 *such that*

$$\sigma_{[\frac{k}{2}]}^{p/q}\rho_k\le c_1 b_k,$$

for every positive integer k;

2)

$$\sup_{n\ge 0}\Big(\sum_{k=0}^{n}a_k\Big)^{p/q}\Big(\sum_{k=n+1}^{\infty}\frac{\big(\rho_k^{-\gamma p}b_k\big)^{1-p'}}{k^{(1-\gamma)p'}}\Big)^{p-1}<\infty,$$

then there exists a positive constant $c>0$, *such that for every* $\{\beta_k\}$ (1) *holds.*

Theorem 4. *Let* $0<\gamma<1$, $1<p<\frac{1}{\gamma}$ *and* $\frac{1}{p}-\frac{1}{q}=\gamma$. *Assume that* $\{\sigma_k\}$ *and* $\{u_k\}$ *be positive, decreasing on* Z^+ *sequences,* $\{\rho_k\}$ *be a positive on* Z^+ *sequence such that* $\{\rho_{|k|}\}\in A_{1+\frac{q}{p'}}(Z)$, $a_k=\sigma_k\rho_k$ *and* $b_k=u_k\rho_k$.

Then the inequality (1) *holds if and only if*

$$\sup_{n\ge 0}\Big(\sum_{k=0}^{n}a_k\Big)^{p/q}\Big(\sum_{k=n+1}^{\infty}\frac{(\rho_k^{-\gamma p}b_k)^{1-p'}}{k^{(1-\gamma)p'}}\Big)^{p-1}<\infty.$$

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