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On Polynomials with Respect to Some Orthogomal Systems

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Let $\phi = \{\varphi_0(x), \varphi_1(x), \dots, \varphi_n(x), \dots\}$, $\varphi_0(x) \equiv 1$, be an orthonortmalized on $[0,1]$ system of functions and μE be the Lebesgue measure of the set $E \subset [0,1]$. By

$$S_m(x) = \sum_{n=1}^{m} a_n \varphi_n(x)$$

we denote the polynomial with respect to the system ϕ (without free term a_0).

The following statements are valid:

Theorem 1. *If* $\sum\limits_{n=1}^{m} a_n^2 > 0$, *then*

$$\mu\Big\{x \in [0,1] : S_m(x) > 0\Big\} > 0$$

and

$$\mu\Big\{x \in [0,1] : S_m(x) < 0\Big\} > 0.$$

Theorem 2. *If ϕ is a full system, then for any number $\varepsilon > 0$ there exists the polynomial* $\sum\limits_{n=1}^{m} b_n \varphi_n(x)$ *such that*

$$\mu\Big\{x \in [0,1] : \sum_{n=1}^{m} b_n \varphi_n(x) > 0\Big\} > 1 - \varepsilon.$$

Note that if the requirements on $\varphi_0(x) \equiv 1$ and on the fullness of system ϕ are not fulfilled, the above theorems are invalid.

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