Mem. Differential Equations Math. Phys. 5(1995), 122–124

A. Lomtatidze and N. Partsvania

ON OSCILLATORY PROPERTIES OF SECOND ORDER SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

(Reported on May 8, 1995)

Consider the system

$$\begin{aligned} u' &= p(t)v, \\ v' &= -q(t)u, \end{aligned} \tag{1}$$

where $p, q : [0, +\infty[\to [0, +\infty[$ are locally summable functions.

Definition. The system (1) is said to be oscillatory if it has at least one oscillatory solution and nonoscillatory otherwise.

It is known (see [1]) that if

$$\int^{+\infty} p(s)\,ds = +\infty \quad \text{and} \quad \int^{+\infty} q(s)\,ds = +\infty,$$

then the system (1) is oscillatory, but if

$$\int^{+\infty} p(s)\,ds < +\infty \quad \text{and} \quad \int^{+\infty} q(s)\,ds < +\infty,$$

then it is nonoscillatory.

Therefore we will assume that

$$\int^{+\infty} p(s)\,ds = +\infty \quad \text{and} \quad \int^{+\infty} q(s)\,ds < +\infty. \tag{2}$$

(The case where $\int^{+\infty} p(s)\,ds < +\infty$ and $\int^{+\infty} q(s)\,ds = +\infty$ can be easily reduced to the case (2)).

As shown in [2], if the condition (2) is fulfilled and for some $\lambda < 1$

$$\int^{+\infty} f^{\lambda}(s)q(s)\,ds = +\infty,$$

where

$$f(t) = \int_0^t p(s)\,ds, \tag{3}$$

1991 *Mathematics Subject Classification.* 34C10.

Key words and phrases. Second order system of ordinary differential equations, oscillatory solution.

then the system (1) is oscillatory.

Therefore we will consider the case where

$$\int^{+\infty} p(s)\,ds = +\infty \quad \text{and} \quad \int^{+\infty} f^{\lambda}(s)q(s)\,ds < +\infty \ \text{ for all } \ \lambda < 1,$$

where f is the function defined by (3).

Introduce the notation

$$g_*(\lambda) = \liminf_{t\to+\infty} f^{1-\lambda}(t) \int_t^{+\infty} f^{\lambda}(s)q(s)\,ds,$$

$$g^*(\lambda) = \limsup_{t\to+\infty} f^{1-\lambda}(t) \int_t^{+\infty} f^{\lambda}(s)q(s)\,ds$$

for $\lambda < 1$ and

$$g_*(\lambda) = \liminf_{t\to+\infty} f^{1-\lambda}(t) \int_0^{t} f^{\lambda}(s)q(s)\,ds,$$

$$g^*(\lambda) = \limsup_{t\to+\infty} f^{1-\lambda}(t) \int_0^{t} f^{\lambda}(s)q(s)\,ds$$

for $\lambda > 1$.

Theorem 1. *Let* $g_*(0) \le \frac{1}{4}$, $g_*(2) \le \frac{1}{4}$ *and*

$$g^*(0) > g_*(0) + \frac{1}{2}\Big(\sqrt{1-4g_*(0)} + \sqrt{1-4g_*(2)}\,\Big).$$

Then the system (1) *is oscillatory.*

Theorem 2. *Let* $g_*(0) \le \frac{1}{4}$ *and* $g_*(2) \le \frac{1}{4}$. *Moreover, let either*

$$g^*(\lambda) > \frac{\lambda^2}{4(1-\lambda)} + \frac{1}{2}\Big(1 + \sqrt{1-4g_*(2)}\,\Big)$$

for some $\lambda < 1$ *or*

$$g^*(\lambda) > \frac{\lambda^2}{4(\lambda-1)} - \frac{1}{2}\Big(1 - \sqrt{1-4g_*(0)}\,\Big)$$

for some $\lambda > 1$. *Then the system* (1) *is oscillatory.*

Corollary 1. *Let*

$$\lim_{\lambda\to1} |1-\lambda| g^*(\lambda) > \frac{1}{4}.$$

Then the system (1) *is oscillatory.*

Corollary 2. *Let for some* $\lambda \ne 1$

$$|1-\lambda| g_*(\lambda) > \frac{1}{4}.$$

Then the system (1) *is oscillatory.*

Theorem 3. *If for some* $\lambda \neq 1$

$$|1-\lambda| g_*(\lambda) > \frac{(2\lambda-1)(3-2\lambda)}{4}, \quad |1-\lambda| g^*(\lambda) < \frac{1}{4},$$

then the system (1) *is nonoscillatory.*

References

1. J. D. Mirzov, Asymptotic behaviour of solutions of systems of nonlinear non-autonomous ordinary differential equations. (Russian) *Maĭkop,* 1993.

Authors' addresses:

A. Lomtatidze
N. Muskhelishvili Institute of
Computational Mathematics
Georgian Academy of Sciences
8, Akuri St., Tbilisi 380093
Georgia

N. Partsvania
A. Razmadze Mathematical Institute
Georgian Academy of Sciences
1, M. Aleksidze St., Tbilisi 380093
Georgia