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Potential Methods for Anisotropic Pseudo-Maxwell Equations in Screen Type Problems

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Dedicated to our friend and colleague Vladimir Rabinovich on the occasion of his 70th birthday anniversary

Abstract. We investigate the Neumann type boundary value problems for anisotropic pseudo-Maxwell equations in screen type problems. It is shown that the problem is well posed in tangent Sobolev spaces and unique solvability and regularity results are obtained via potential methods and the coercivity result of Costabel on the bilinear form associated to pseudo-Maxwell equations.

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1. Introduction

The purpose of the present paper is to investigate the screen-type boundary value problem for pseudo-Maxwell equations

$$\operatorname{curl} \mu^{-1} \operatorname{curl} \boldsymbol{U} - s\, \varepsilon \operatorname{grad} \operatorname{div} (\varepsilon \boldsymbol{U}) - \omega^2 \varepsilon \boldsymbol{U} = 0 \quad \text{in} \quad \Omega, \tag{1.1}$$

where Ω is a bounded or an unbounded domain with boundary, using the potential method.

The present investigation covers the anisotropic case when the matrices

$$\varepsilon = [\varepsilon_{jk}]_{3\times 3}, \qquad \mu = [\mu_{jk}]_{3\times 3} \tag{1.2}$$

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in (1.1) are real valued, constant, symmetric and positive definite, i.e.,

$$\langle\varepsilon\xi,\xi\rangle \geq c|\xi|^2\,, \qquad \langle\mu\xi,\xi\rangle \geq d|\xi|^2\,, \qquad \forall\xi\in\mathbb{R}^3\,,$$

for some positive constants $c>0$, $d>0$, where

$$\langle\eta,\xi\rangle := \sum_{j=1}^{3}\eta_j\overline{\xi}_j, \quad \eta,\ \xi\in\mathbb{C}^3.$$

s is a positive real number and the frequency parameter ω is assumed to be non-zero and complex valued, i.e., $\operatorname{Im}\omega\neq 0$.

The study of boundary value problems in electromagnetism naturally leads us to the pseudo-Maxwell equations inherited with tangent boundary conditions, which are in some sense non-standard for the elliptic equations (1.1), cf. works of Buffa, Costabel, Christiansen, Dauge, Hazard, Lenoir, Mitrea, Nicaise and others. The case with the Dirichlet type boundary condition $\boldsymbol{\nu}\times\boldsymbol{U}$ is mostly investigated by variational methods, here $\boldsymbol{\nu}$ is the unit normal to the boundary $\partial\Omega$. Our goal is to investigate well-posedness of the Neumann type boundary value problems for (1.1) as well as its unique solvability in unbounded domains with screen configuration, i.e.,

$$\Omega=\mathbb{R}^3_{\mathcal{C}} := \mathbb{R}^3\setminus\overline{\mathcal{C}},$$

where $\mathcal{C}$ denotes a smooth open hypersurface with a smooth boundary.

2. Neumann boundary value problems for pseudo-Maxwell equations

From now on throughout the paper, unless stated otherwise, Ω denotes either a bounded $\Omega^+\subset\mathbb{R}^3$ or an unbounded $\Omega^- := \mathbb{R}^3\backslash\overline{\Omega^+}$ domain with smooth boundary $\mathcal{S} := \partial\Omega^+$ and $\boldsymbol{\nu}$ is the outer unit normal vector field to $\mathcal{S}$. Whenever necessary, we will specify the case.

For rigorous formulation of conditions for the unique solvability of the formulated boundary value problems we use the Bessel potential $\mathbb{H}^r(\Omega)$, $\mathbb{H}^r(\mathcal{S})$ spaces. We quote [20] for definitions and properties of these spaces.

By $\mathcal{C}$ we denote an orientable smooth open surface in $\mathbb{R}^3$ (a screen) with boundary $\partial\mathcal{C}$, which has two faces $\mathcal{C}^-$ and $\mathcal{C}^+$ distinguished by the orientation of the normal vector field: $\boldsymbol{\nu}$ is pointing from $\mathcal{C}^+$ to $\mathcal{C}^-$. Moreover, we assume that $\mathcal{C}$ is a part of some smooth and simple (non self intersecting) hypersurface $\mathcal{S}$ that divides the space $\mathbb{R}^3$ into two disjoint domains Ω^+ and $\Omega^- := \mathbb{R}^3\backslash\overline{\Omega^+}$ such that Ω^+ is bounded and $\mathcal{S}=\partial\Omega^\pm$.

The space $\widetilde{\mathbb{H}}^r(\mathcal{C})$ comprises those functions $\varphi\in\mathbb{H}^r(\mathcal{S})$ which are supported in $\overline{\mathcal{C}}$ (functions with the "vanishing traces on the boundary"). For the detailed definitions and properties of these spaces we refer, e.g., to [13, 14, 20]).

We did not distinguish notation for the Banach spaces and their vector analogues unless this does not lead to a confusion. Although we use the boldface