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ANALYSIS OF DIRECT BOUNDARY-DOMAIN INTEGRAL EQUATIONS FOR A MIXED BVP WITH VARIABLE COEFFICIENT, I: EQUIVALENCE AND INVERTIBILITY

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ABSTRACT. A mixed (Dirichlet-Neumann) boundary value problem (BVP) for the "stationary heat transfer" partial differential equation with variable coefficient is reduced to some systems of nonstandard segregated direct parametrix-based boundary-domain integral equations (BDIEs). The BDIE systems contain integral operators defined on the domain under consideration as well as potential-type and pseudo-differential operators defined on open submanifolds of the boundary. It is shown that the BDIE systems are equivalent to the original mixed BVP, and the operators of the BDIE systems are invertible in appropriate Sobolev spaces.

1. Introduction. The boundary integral equation (BIE) method has been intensively developed over recent decades both in theory and in engineering applications. Its popularity is due to the possibility of reducing a boundary value problem (BVP) for a partial differential equation in a domain to an integral equation on the boundary of the domain. This approach diminishes the problem dimensionality by one which is very important for construction of various numerical algorithms requiring small computer resources. The main ingredient necessary for reduction of a BVP to a boundary integral equation (BIE) is a fundamental solution to the original partial differential equation, available in an analytical form and/or cheaply calculated. After the

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