LOCALIZATION AND MINIMAL NORMALIZATION OF SOME BASIC MIXED BOUNDARY VALUE PROBLEMS

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To Professor Georgii S. Litvinchuk on the occasion of his 70th birthday

Abstract We consider a class of mixed boundary value problems in spaces of Bessel potentials. By localization, an operator L associated with the BVP is related through operator matrix identities to a family of pseudodifferential operators which leads to a Fredholm criterion for L. But particular attention is devoted to the non-Fredholm case where the image of L is not closed. Minimal normalization, which means a certain minimal change of the spaces under consideration such that either the continuous extension of L or the image restriction, respectively, is normally solvable, leads to modified spaces of Bessel potentials. These can be characterized in a physically relevant sense and seen to be closely related to operators with transmission property (domain normalization) or to problems with compatibility conditions for the data (image normalization), respectively.

Keywords: Normalization, boundary value problems, localization, pseudo-differential operators, Wiener-Hopf operators, Fredholm property, Bessel potential spaces.

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1. Introduction to mixed boundary value problems and normalization

We confine our attention to the following model boundary value problem (BVP) based on considerations in [38, p. 186 ff.] and [37]. Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary $\Gamma = \partial\Omega$ divided into two simply connected parts and their common boundary points, i.e. (see Figure 1.1)

$$\Gamma = \Gamma^1 \cup \Gamma^2 \cup \{x^1, x^2\}. \tag{1.1}$$

Let A be a linear differential operator with smooth coefficients in Ω of order $2m$ where $m \in \mathbb{N}_1$ and B^1, B^2 are vectors of linear boundary operators both with smooth coefficients on Γ (extendible to $\overline{\Omega}$) of order $m^1 = (m_1^1, \dots, m_m^1)$ and $m^2 = (m_1^2, \dots, m_m^2)$, respectively, such that $0 \leq m_j^1, m_j^2 \leq 2m-1$. More precisely we have B^k with components

$$b_j^k = b_j^k(x, D) = \sum_{|s| \leq m_j^k} b_{j,s}^k(x) T_0^k(D^s\varphi) = T_0^k\left(\sum_{|s| \leq m_j^k} b_{j,s}^k(x) D^s\varphi\right), \quad k = 1, 2 \tag{1.2}$$

where T_0^k denotes the (usual) trace operator on Γ^k.

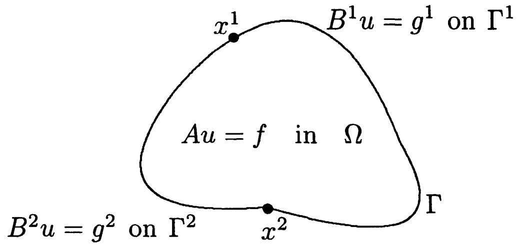

Figure 1.1: Mixed boundary value problem.

We look for all solutions $u \in H^{2m+l}(\Omega)$, $l \geq 0$, such that

$$\begin{array}{rcll} Au(x) & = & A(x, D)u(x) \;=\; f(x), & x \in \Omega \\ B^k u(x) & = & \left(b_1^k(x, D)u(x), \dots, b_m^k(x, D)u(x)\right) & \\ & = & \left(g_1^k(x), \dots, g_m^k(x)\right), & x \in \Gamma^k, \quad k = 1, 2, \end{array} \tag{1.3}$$

where $f \in H^l(\Omega)$, $g_j^k \in H^{2m+l-m_j^k-1/2}(\Gamma^k)$ are (arbitrarily) given, $j = 1, \dots, m$, and refer, for short, to the *mixed BVP* (1.3). It is called *piecewise elliptic*, if B^k $(k = 1, 2)$ have extensions $\widetilde{B^k}$ to the whole Γ such that (1.3) with B^k replaced