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Abstract. At the filling factor ν=2, the bilayer quantum Hall system has three phases, the spin-ferromagnet phase, the spin singlet phase and the canted antiferromagnet (CAF) phase, depending on the relative strength between the Zeeman energy and interlayer tunneling energy. We present a systematic method to derive the effective Hamiltonian for the Goldstone modes in these three phases. We then investigate the dispersion relations and the coherence lengths of the Goldstone modes. To explore a possible emergence of the interlayer phase coherence, we analyze the dispersion relations in the limit of zero tunneling energy. We find one gapless mode with the linear dispersion relation in the CAF phase.

1. Introduction

In the bilayer quantum Hall (QH) system, at total Landau filling factor $\nu = 2$, a rich phase structure emerges by the interplay between the spin and the layer (pseudospin) degrees of freedom [1, 2]. According to the one-body picture we expect to have two phases depending on the relative strength between the Zeeman gap $\Delta_{\rm Z}$ and the tunneling gap $\Delta_{\rm SAS}$. One is the spin-ferromagnet and pseudospin-singlet phase (abridged as the spin phase) for $\Delta_{\rm Z} > \Delta_{\rm SAS}$; the other is the spin-singlet and pseudospin ferromagnet phase (abridged as the pseudospin phase) for $\Delta_{\rm SAS} > \Delta_{\rm Z}$. Instead, an intermediate phase, a canted antiferromagnetic phase (abridged as the CAF phase) emerges. This is a novel phase where the spin direction is canted and make antiferromagnetic correlations between the two layers[2].

The ground state structure of the $\nu = 2$ bilayer QH system has been investigated based on the SU(4) formalism [3, 4]. However, the effective Hamiltonian for the Goldstone modes has not been derived, especially in the CAF phase. On the other hand, experimentally, a role of a Goldstone mode has been suggested by nuclear magnetic resonance[5] in the CAF phase.

In this paper we develop a generic formalism to determine the symmetry breaking pattern and to derive the effective Hamiltonian for the Goldstone modes in the three phases of the $\nu = 2$ bilayer QH system. The symmetry breaking pattern reads, $\mathrm{SU}(4) \to \mathrm{U}(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)$, and there appear eight Goldstone modes in each phase. The corresponding Goldstone modes in the two phases match smoothly at the phase boundary.

This paper is organized as follows. In Sec. 2, we review the Coulomb interaction of the bilayer QH system projected to the lowest Landau level (LLL) and the SU(4) effective Hamiltonian

after making the derivative expansion. We also review the ground state structure in the three phases. In Sec. 3, which is the main part of this paper, we develop a unified formalism to derive the effective Hamiltonian for the Goldstone modes. Then we discuss the SU(4) symmetry breaking pattern, the dispersion relations, and the coherence length in each phase. We study the dispersions and the coherence length in the limit $\Delta_{\rm SAS} \to 0$, to explore a possible emergence of the interlayer phase coherence in the CAF phase. Remarkably, we find one gapless mode with the linear dispersion relation. Section 4 is devoted to discussion.

2. The SU(4) Effective Hamiltonian and the Ground State Structure

In the bilayer system an electron has two types of indices, the spin index $(\uparrow, \downarrow)$ and the layer index (f, b). They can be incorporated in 4 types of isospin index $\alpha = \mathrm{f}\uparrow, \mathrm{f}\downarrow, \mathrm{b}\uparrow, \mathrm{b}\downarrow$. The electron field $\psi_\alpha(\boldsymbol{x})$ has four components, and the bilayer system possesses the underlying algebra SU(4) with having the subalgebra $\mathrm{SU}_{\rm spin}(2) \times \mathrm{SU}_{\rm ppin}(2)$. We denote the three generators of the $\mathrm{SU}_{\rm spin}(2)$ by $\tau_a^{\rm spin}$, and those of $\mathrm{SU}_{\rm ppin}(2)$ by $\tau_a^{\rm ppin}$. There are remaining nine generators $\tau_a^{\rm spin}\tau_b^{\rm ppin}$. Their explicit form is given in Appendix D in Ref.[1].

All the physical operators required for the description of the system are constructed as the bilinear combinations of $\psi(\boldsymbol{x})$ and $\psi^\dagger(\boldsymbol{x})$. There are 16 density operators

$$\begin{aligned} \rho(\boldsymbol{x}) &= \psi^\dagger(\boldsymbol{x})\psi(\boldsymbol{x}), \quad S_a(\boldsymbol{x}) = \frac{1}{2}\psi^\dagger(\boldsymbol{x})\tau_a^{\rm spin}\psi(\boldsymbol{x}), \\ P_a(\boldsymbol{x}) &= \frac{1}{2}\psi^\dagger(\boldsymbol{x})\tau_a^{\rm ppin}\psi(\boldsymbol{x}), \quad R_{ab}(\boldsymbol{x}) = \frac{1}{2}\psi^\dagger(\boldsymbol{x})\tau_a^{\rm spin}\tau_b^{\rm ppin}\psi(\boldsymbol{x}), \end{aligned} \tag{1}$$

where S_a describes the total spin, $2P_z$ measures the electron-density difference between the two layers. The operator R_{ab} transforms as a spin under $\mathrm{SU}_{\rm spin}(2)$ and as a pseudospin under $\mathrm{SU}_{\rm ppin}(2)$.

The kinetic Hamiltonian is quenched, since the kinetic energy is common to all states in the LLL. The Coulomb interaction is decomposed into the SU(4)-invariant $H_{\rm C}^+$ and SU(4)-noninvariant terms $H_{\rm C}^-$. The tunneling and bias terms are summarized into the pseudo-Zeeman term. Combining the Zeeman and pseudo-Zeeman terms we have

$$H_{\rm ZpZ} = -\int d^2x (\Delta_{\rm Z} S_z + \Delta_{\rm SAS} P_x + \Delta_{\rm bias} P_z), \tag{2}$$

with the Zeeman gap $\Delta_{\rm Z}$, the tunneling gap $\Delta_{\rm SAS}$, and the bias voltage $\Delta_{\rm bias} = eV_{\rm bias}$. The total Hamiltonian is $H = H_{\rm C}^+ + H_{\rm C}^- + H_{\rm ZpZ}$.

We project the density operators (1) to the LLL. What are observed experimentally are the classical densities, which are expectation values such as $\rho^{\rm cl}(\boldsymbol{p}) = \langle \mathfrak{S}|\rho(\boldsymbol{p})|\mathfrak{S}\rangle$, where $|\mathfrak{S}\rangle$ represents a generic state in the LLL. The SU(4) effective Hamiltonian density are given by[3]

$$\begin{aligned} \mathcal{H}^{\rm eff} = {} & J_s^d \left(\sum (\partial_k \mathcal{S}_a)^2 + (\partial_k \mathcal{P}_a)^2 + (\partial_k \mathcal{R}_{ab})^2 \right) + 2J_s^- \left(\sum (\partial_k \mathcal{S}_a)^2 + (\partial_k \mathcal{P}_z)^2 + (\partial_k \mathcal{R}_{az})^2 \right) \\ & + \rho_\phi [\epsilon_{\rm cap} (\mathcal{P}_z)^2 - 2\epsilon_X^- \left(\sum (\mathcal{S}_a)^2 + (\mathcal{R}_{az})^2 \right) - (\epsilon_X^+ - \epsilon_X^-)(\sum \left(\mathcal{S}_a \right)^2 + (\mathcal{P}_a)^2 + (\mathcal{R}_{ab})^2) \\ & - (\Delta_{\rm Z} \mathcal{S}_z + \Delta_{\rm SAS} \mathcal{P}_x + \Delta_{\rm bias} \mathcal{P}_z) - (\epsilon_X^+ + \epsilon_X^-)], \end{aligned} \tag{3}$$

where J_s, J_s^d, $\epsilon_{\rm cap}$ and $\epsilon_X^\pm$, are the intralyer stiffness, the interlayer stiffness, the capacitance energy and the exchange Coulomb energy, respectively, with the explicit formula given in Appendix A in Ref.[1]. $\rho_\Phi = \rho_0/\nu$ is the density of states, and we set $\rho^{\rm cl}(\boldsymbol{p}) = \rho_0$, $S_a^{\rm cl}(\boldsymbol{p}) = \rho_\Phi \mathcal{S}_a(\boldsymbol{p})$, $P_a^{\rm cl}(\boldsymbol{p}) = \rho_\Phi \mathcal{P}_a(\boldsymbol{p})$, and $R_{ab}^{\rm cl}(\boldsymbol{p}) = \rho_\Phi \mathcal{R}_{ab}(\boldsymbol{p})$ for the study of Goldstone modes.

It is to be remarked that all potential terms vanish in the SU(4) invariant limit, where perturbative excitations are gapless. They are the Goldstone modes associated with spontaneous breaking of the SU(4) symmetry. They get gapped in the actual system, since the SU(4) symmetry is explicitly broken. Nevertheless we call them the Goldstone modes.

The ground state is obtained by minimizing the effective Hamiltonian (3) for homogeneous configurations of the classical densities. The order parameters are the classical densities for the ground state. It has been shown[4] at $\nu = 2$ that they are given as

$$\mathcal{S}_z^0 = \frac{\Delta_{\rm Z}}{\Delta_0}(1-\alpha^2)\sqrt{1-\beta^2}, \quad \mathcal{P}_x^0 = \frac{\Delta_{\rm SAS}}{\Delta_0}\alpha^2\sqrt{1-\beta^2}, \quad \mathcal{P}_z^0 = \frac{\Delta_{\rm SAS}}{\Delta_0}\alpha^2\beta,$$
$$\mathcal{R}_{xx}^0 = -\frac{\Delta_{\rm SAS}}{\Delta_0}\alpha\sqrt{1-\alpha^2}\beta, \quad \mathcal{R}_{yy}^0 = -\frac{\Delta_{\rm Z}}{\Delta_0}\alpha\sqrt{1-\alpha^2}\sqrt{1-\beta^2}, \quad \mathcal{R}_{xz}^0 = \frac{\Delta_{\rm SAS}}{\Delta_0}\alpha\sqrt{1-\alpha^2}\sqrt{1-\beta^2}, \tag{4}$$

with $\Delta_0 = \sqrt{\Delta_{\rm SAS}^2\alpha^2 + \Delta_{\rm Z}^2(1-\alpha^2)(1-\beta^2)}$ and all the rest components are zero. The parameters α and β, satisfy the condition, $|\alpha| \leq 1$ and $|\beta| \leq 1$. As a physical variable it is convenient to use the imbalance parameter $\sigma_0 \equiv \mathcal{P}_z^0$, instead of the bias voltage $\Delta_{\rm bias}$. This is possible in the pseudospin and CAF phases. The bilayer system is balanced at $\sigma_0 = 0$, while all electrons are in the front layer at $\sigma_0 = 1$, and in the back layer at $\sigma_0 = -1$. There are three phases, and we discuss them in terms of α and β.

First, when $\alpha = 0$, it follows that

$$\mathcal{S}_z^0 = 1, \tag{5}$$

and all others being zero. This is the spin phase, which is characterized by the fact that the isospin is fully polarized into the spin direction. The spins in both layers point to the positive z axis due to the Zeeman effect.

Second, when $\alpha = 1$, we have

$$\mathcal{P}_x^0 = \sqrt{1-\beta^2}, \qquad \mathcal{P}_z^0 = \beta = \sigma_0, \tag{6}$$

and all the others being zero. This is the pseudospin phase, which is characterized by the fact that the isospin is fully polarized into the pseudospin direction as given by (6).

For intermediate values of α ($0 < \alpha < 1$), not only the spin and pseudospin but also some residual components are nonvanishing, where we may control the density imbalance by applying a bias voltage as in the pseudospin phase. It follows from (4) that, as the system goes away from the spin phase ($\alpha = 0$), the spins begin to cant coherently and make antiferromagnetic correlations between the two layers.

The interlayer phase coherence is an intriguing phenomenon in the bilayer QH system[1]. Since it is enhanced in the limit $\Delta_{\rm SAS} \to 0$, it is worthwhile to investigate the effective Hamiltonian at $\nu = 2$ in this limit. We need to know how the parameters α and β are expressed in terms of the physical variables. Up to the order $O(\Delta_{\rm SAS}^2)$, the solutions are

$$\beta = \pm\sqrt{1-\left(\frac{\Delta_{\rm SAS}}{\Delta_{\rm Z}}\right)^2} + O(\Delta_{\rm SAS}^4), \tag{7}$$

with $\Delta_0 \to \Delta_{\rm SAS} + O(\Delta_{\rm SAS}^3)$. Then we have $\mathcal{P}_z^0 = \sigma_0 = \pm\alpha^2 + O(\Delta_{\rm SAS}^2)$. The parameters α and β are simple functions of the physical variables $\Delta_{\rm SAS}/\Delta_{\rm Z}$ and σ_0 in the limit $\Delta_{\rm SAS} \to 0$. We might expect novel phenomena associated with the interlayer phase coherence in the CAF phase.

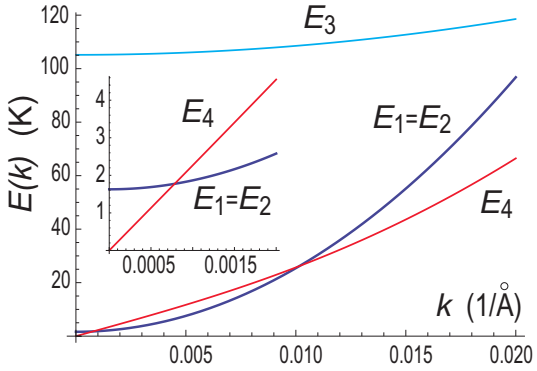

Figure 1. Dispersion relations (10) for the four Goldstone modes E_i. The sample parameters are $d = 231\mathring{A}$, B=5.5T, $\rho_0 = 2.7 \times 10^{15}\mathrm{m}^{-2}$, and $\alpha = 1/\sqrt{2}$. Inset: Dispersion relations near $k = 0$. It is clear that $E_4(k)$ is linear.

3. Effective Hamiltonian for Goldstone Modes

Having reviewed the three phases in the bilayer system at $\nu = 2$, we proceed to discuss the symmetry breaking pattern and construct the effective Hamiltonian for the Goldstone modes in each phase. There is a systematic method for this purpose[6].

We analyze excitations around the classical ground state (4). It is convenient to introduce the SU(4) isospin notation such that, $\mathcal{I}^{(0)}_{a0} = \mathcal{S}^0_a$, $\mathcal{I}^{(0)}_{0a} = \mathcal{P}^0_a$, $\mathcal{I}^{(0)}_{ab} = \mathcal{R}^0_{ab}$. We set all of them into one 15-dimensional vector $\mathcal{I}^{(0)}_{\mu\nu}$ with the index $\mu\nu$: Note that there is no component $\mathcal{I}^{(0)}_{00}$. We parametrize the SU(4) isospin operators as

$$\mathcal{I}_{\mu\nu}(x) = \left[\exp\left(i\sum_{\gamma\delta}\pi_{\gamma\delta}T_{\gamma\delta}\right)\right]^{\mu'\nu'}_{\mu\nu}\mathcal{I}^0_{\mu'\nu'}, \tag{8}$$

where $T_{\gamma\delta}$ are the matrices of the broken SU(4) generators in the adjoint representation of SU(4), each of which is a 15×15 matrix. The greek indices run over $0, x, y, z$. The phase field $\pi_{\gamma\delta}(x)$ are the eight Goldstone modes associated with the broken generators, and hence, only eight generating matrices $T_{\gamma\delta}$ are involved in the formula (8). Then we may identify $\mathcal{S}_a = \mathcal{I}_{a0}$, $\mathcal{P}_a = \mathcal{I}_{0a}$, $\mathcal{R}_{ab} = \mathcal{I}_{ab}$, and express various physical variables in terms of the Goldstone modes $\pi_{\gamma\delta}(x)$. We then expand the formula (8) in $\pi_{\gamma\delta}$ as, $\mathcal{I}_{\mu\nu}(x) = \mathcal{I}^0_{\mu\nu} + \mathcal{I}^{(1)}_{\mu\nu}(x) + \mathcal{I}^{(2)}_{\mu\nu}(x) + \cdots$, where $\mathcal{I}^{(n)}_{\mu\nu}(x)$ is the nth order term in the Goldstone mode $\pi_{\gamma\delta}$. Each phase is characterized by the order parameter $\mathcal{I}^0_{\mu\nu}$, which are nothing but (4). The key observation is that the first order term $\mathcal{I}^{(1)}_{\mu\nu}(x)$ contains all informations about the symmetry breaking pattern and the associated Goldstone modes, yielding their kinematic terms. On the other hand, the second order term $\mathcal{I}^{(2)}_{\mu\nu}(x)$ provides them with gaps. For the detailed discussions, see Ref.[7].

3.1. CAF Phase in $\Delta_{SAS} \to 0$

The effective Hamiltonian in the CAF phase is too complicated. We take the limit $\Delta_{\mathrm{SAS}} \to 0$ to examine if some simplified formulas are obtained. In particular we would like to seek for gapless modes. Such gapless modes will play an important role to drive the interlayer coherence in the

CAF phase. In this limit we have the Hamiltonian

$$H = \sum_{i=1}^{4} \int d^2k E_i \eta^\dagger_{i,\boldsymbol{k}} \eta_{i,\boldsymbol{k}}, \tag{9}$$

together with the dispersion relations (Figure 1)

$$E_1 = E_2 = \frac{4}{\rho_0}((1-\alpha^2)J_s + \alpha^2 J_s^d)\boldsymbol{k}^2 + \Delta_{\rm Z}, \quad E_3 = \frac{4\boldsymbol{k}^2}{\rho_0} J_s^d + 2\Delta_{\rm Z} + 8\cos^2\theta_\alpha \epsilon_X^-,$$
$$E_4 = |\boldsymbol{k}| \sqrt{\frac{8J_s^d}{\rho_0}\left(\frac{2\boldsymbol{k}^2}{\rho_0}(\cos^2 2\theta_\alpha J_s^d + \sin^2 2\theta_\alpha J_s) + 2\sin^2 2\theta_\alpha(\epsilon_D^- - \epsilon_X^-)\right)}, \tag{10}$$

where $\cos\theta_\alpha = \sqrt{1-\alpha^2}$, $\sin\theta_\alpha = \alpha$, and $\eta_{i,\boldsymbol{k}}$ $(i = 1,2,3,4)$ are the annihilation operators satisfying the commutation relation $\left[\eta_{i,\boldsymbol{k}}, \eta^\dagger_{j,\boldsymbol{k}'}\right] = \delta_{ij}\delta(\boldsymbol{k}-\boldsymbol{k}')$.

We summarize the Goldstone modes in the CAF phase in the limit $\Delta_{\rm SAS} \to 0$. It is to be emphasized that there emerges one gapless mode, $\eta_{4,\boldsymbol{k}}$, reflecting the realization of an exact and its spontaneous breaking of a U(1) part of the SU(4) rotational symmetry. Furthermore, it has the linear dispersion relation as in (10), as leads to a superfluidity associated with this gapless mode. All other modes have gaps.

4. Discussion

We have presented a systematic method based on the formula (8) to investigate the symmetry breaking pattern and to derive the effective Hamiltonian for the Goldstone modes in the ν=2 bilayer QH system, particularly, for the CAF phase. Eight Goldstone modes emerge in each phase, which are shown to be smoothly transformed one to another across the phase boundary.

The interlayer phase coherence and the Josephson effect are among the most intriguing phenomena in the $\nu = 1$ bilayer QH system[1]. They are enhanced in the limit $\Delta_{\rm SAS} \to 0$. It is natural to seek for similar phenomena in the $\nu = 2$ bilayer QH system in the CAF phase, where the electron densities can be controlled arbitrarily in both layers. Having investigated the dispersion relations and the coherence length in the limit $\Delta_{\rm SAS} \to 0$, remarkably, we have found one coherent mode whose coherence length diverges. Furthermore it has the linear dispersion relation. It might be responsible to the interlayer phase coherence.

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