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ON THE FUBINI'S SETS OF MULTIPLE FUNCTION SERIES

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Let $d \geq 2$ be some natural number, R^d the Euclidean space of dimension d, Z_0^d a set of points from R^d with integer nonnegative coordinates. By $x = (x_1, \dots, x_d)$ we denote the points from the unit cube $I^d = [0,1]^d$ and by $m = (m_1, \dots, m_d)$ and $n = (n, \dots, n_d)$ those from the set z^d. $n \geq 0$ means that $n_j \geq 0$, $j = 1, 2, \dots d$. The symbol $m \to \infty$ implies that $m_j \to \infty$ for every j, $1 \leq j \leq d$ independently of each other. μ is the linear Lebesgue measure. $E_1 \times E_2$ is the Cartesian product of the sets E_1 and E_2.

Let on $[0,1]$ be given a system of measurable functions $\phi = \{\varphi_i(t)\}_{i=0}^{\infty}$,

$$|\varphi_i(t)| < \infty, \quad t \in [0,1], \quad i = 0, 1, 2, \dots.$$

For every $n \in Z_0^d$ we denote

$$\phi_n(x) = \prod_{j=1}^{d} \varphi_{n_j}(x_j), \quad x \in [0,1]^d.$$

Consider the d-multiple series with respect to the system $\phi^d = \{\phi_n(x)\}_{n \geq 0}$,

$$\sum_{n=0}^{\infty} a_n \phi_n(x) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d}^{\infty} a_{n_1,\dots,n_d} \prod_{j=1}^{d} \varphi_{n_j}(x_j). \tag{1}$$

By $S_m(x)$ we denote rectangular partial sums of the series (1), i.e.,

$$S_m(x) = \sum_{n_1=0}^{m_1} \cdots \sum_{n_d=0}^{m_d} a_{n_1,\dots,n_d} \prod_{j=1}^{d} \varphi_{n_j}(x_j).$$

The convergence of the series (1) at the point x implies that there exists a finite Pringsheim limit, i.e.,

$$\lim_{m \to \infty} S_m(x). \tag{2}$$

Let σ be any one-to-one mapping of the set $\{1, 2, \dots, d\}$ onto itself. All possible repeated limits of partial sums $S_m(x)$ we denote by

$$\lim_{m_{\sigma(1)} \to \infty} \Big(\lim_{m_{\sigma(2)} \to \infty} \Big(\dots \Big(\lim_{m_{\sigma(d)} \to \infty} S_m(x) \Big) \dots \Big)\Big) \tag{3}$$

Definition 1. We say that $E \subset [0,1]^d$ is the Fubini's set for the multiple series (1), if the existence of the limit (2) on the set E implyis existence of all possible limits (3) on the set E, and the equalities

$$\lim_{m \to \infty} S_m(x) = \lim_{m_{\sigma(1)} \to \infty} \Big(\lim_{m_{\sigma(2)} \to \infty} \Big(\dots \Big(\lim_{m_{\sigma(d)} \to \infty} S_m(x) \Big) \dots \Big)\Big), \quad x \in E \tag{4}$$

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are fulfilled.

In particular, it is shown in [1] that for any countable set D the set $[0,1]^d \setminus D$ is Fubini's one for multiple trigonometric series.

Definition 2. They say that $\phi = \{\varphi_i(t)\}_{i=0}^{\infty}$ is the system of ε-uniqueness, if there exists a number $\varepsilon \in]0,1]$, such that the convergence of the series

$$\sum_{i=0}^{\infty} a_i \varphi_i(t)$$

to zero on the set $A \subset [0,1]$, $\mu A > 1 - \varepsilon$ results in the relation $a_i = 0$ $(i = 0, 1, 2, \ldots)$.

The following theorem is valid.

Theorem. *Let* $\phi = \{\varphi_i(t)\}_{i=0}^{\infty}$ *be the system of* ε*-uniqueness and for any* j, $1 \le j \le d$ *the sets* $E_j \subset [0,1]$ *are such that* $\mu E_j > 1 - \varepsilon$. *Then*

$$E = E_1 \times E_2 \times \cdots E_d$$

is the Fubini's set for the multiple series (1).

References

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