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STOCHASTIC DERIVATIVE OF POISSON POLYNOMIAL FUNCTIONALS

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Abstract. A new definition of the stochastic derivative operator for Poisson polynomial functionals is introduced, which is not based on the chaotic decomposition of functional, as well as in Ma, Protter and Martin's 1998 work. The equivalence of this two definitions for Poisson functionals is shown and some basic properties of stochastic derivative operator are investigated.

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I. As is known, in the theory of standard integration, the requirement for the integrand to be measurable is a very small restriction as compared to the condition of integrability which implies the boundedness in a certain sense of an absolute integrand value. As for the stochastic Ito's integral $\int_0^T f(t,\omega)dw_t$, the situation here is opposite. Besides the fact that the integrand $f(t,\omega)$ is the measurable function of two variables, it should be the adapted (nonanticipated) process, i.e for any $t\in[0,T]$ the random variable $f(t,\cdot)$ should be measurable with respect to the $\mathcal{F}_t^w:=\sigma\{w_s, s\in[0,t]\}$–$\sigma$-algebra. On the one hand, this requirement is natural for many situations, when filtration shows possible evolution of information. On the other hand, over a long period of time this requirement restricted both the development of the theory and the application of stochastic calculus.

Starting from the 70^{th} of the past century, many attempts were made to weak the requirement for the integrand to be adapted for the integrand of the Ito's stochastic integral as well as in the theory of "the extension of filtration". Skorokhod (1975) suggested absolutely different method, symmetric with respect to the time inversion and did not require for the integrand to be independent of the future Wiener process. Towards this end, he required for the integrand to be smooth in a certain sense, i.e., its stochastic differentiability. This idea was later on developed in the works of Gaveau-Trauber

(1982), Nualart, Zakai (1986), Pardoux (1982), Protter, Malliavin (1979), etc. In particular, Gaveau and Trauber have proved that the Skorokhod operator of stochastic integration coincides with the conjugate operator of a stochastic derivative operator.

Ma, Protter and Martin (1998) have proposed an anticipating integral for the class of so-called normal martingales (a martingale M is called normal if $\langle M, M\rangle_t = t$) which have the chaos representation property. It is analogous to the Skorohod integral as developed by Nualart and Pardoux (1988). When M is Wiener process, it is exactly the Skorohod integral. There are many similarities between the above-mentioned martingale anticipating integral and the Skorohod integral, but there are also some important differences. Many of these differences stem from one key fact: in the Wiener case $[w, w]_t = \langle w, w\rangle_t = t$, while in the normal martingale case only $\langle M, M\rangle_t = t$, and $[M, M]_t$ is random. For example, there are two ways to describe the variational derivative (also known as the Malliavin derivative in the Wiener case), and they are equivalent in the Wiener case but not in the martingale case. In [5] an example is given, which shows that in the martingale case one cannot define the stochastic derivative operator in the usual way to obtain the Sobolev space structure for the space $D_{2,1}$. Indeed, this example shows that the two definitions (Sobolev space and chaos expansion) are compatible if and only if $[M, M]_t$ is deterministic. Therefore in the martingale case the space $D^M_{q,1}, (1 < q < 2)$ cannot be defined in the usual way (i.e., by closing the class of smooth functionals with respect to the corresponding norm).

On the other hand, in the theory of random processes special place take the so-called martingale representation theorems. In the eighties of the past century, it turned out (see Harison and Pliska (1981)) that the martingale representation theorems (along with the Girsanov's measure change theorem) play an important role in the modern financial mathematics. According to the Ocone-Haussmann-Clark formula (see [3]), if $F \in D^M_{2,1}$, then the Ocone-Haussmann-Clark's representation

$$F = EF + \int_0^T {}^p(D^M_t F)dM_t$$

is valid. Here $D^M_{2,1}$ denotes the space of quadratically integrable functionals having the first order stochastic derivative, and ${}^p(D^M_t F)$ is the predictable projection of the stochastic derivative $D^M_t F$ of the functional F. In work of Jaoshvili and Purtukhia (2008) the space $D^M_{q,1}$ $(1 < q < 2)$ for the compensated Poisson process is proposed and the integral representation formula of Ocone and Haussmann-Clark for functionals from this space is established.

II. Let $w_t, t \in [0, 1]$ be a d-dimensional standard Wiener process defined on the canonical probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and $\mathcal{F}_t = \sigma\{w_s, s \in [0, t]\}$.

Let $C^\infty_b(R^k)$ be the set of C^∞ functions $f : R^k \to R^1$ which are bounded and have bounded derivatives of all orders. A smooth functional will be a random variable $F : \Omega \to R^1$ of the form $F = f(w_{t_1}, ...w_{t_n})$, where the function $f(x^{11}, ..., x^{d1}; ...; x^{1n}, ..., x^{dn})$ belongs to $C_b^\infty(R^{dn})$ and $t_1, ..., t_n \in [0, 1]$. The class of smooth functionals will be denoted by **SF**.

The derivative of a smooth functional F can be defined as the d-dimensional stochastic process given by the relation

$$(D_t^w F)^j = \sum_{i=1}^{n} \frac{\partial f}{\partial x^{ji}}(w_{t_1}, ...w_{t_n}) I_{[0,t_i]}(t), \ \ t \in [0, 1], \ j = 1, ..., d.$$

For example, $D_t^w w_s = I_{[0,s]}(t)$. The operator D^w can be considered as an unbounded operator defined on a dense subset of $L_2(\Omega)$ and taking value on $L_2([0, T])$. For any real $p > 1$ we introduce the semi-norm on **SF**:

$$\|F\|_{p,1} := \|F\|_{L_2(\Omega)} + \| \|D_{\cdot}^w F\|_{L_2([0,1])} \|_{L_2(\Omega)}.$$

Let $D_{p,1}^w$ be the Banach space which is the completion of **SF** with respect to the norm $\|\cdot\|_{p,1}$. The space $D_{2,1}^w$ is a Hilbert space with the scalar product

$$\langle F, G \rangle_{2,1} := E(FG) + E[\langle D_{\cdot}^w F, D_{\cdot}^w G \rangle_{L_2([0,1])}].$$

Consider the orthogonal Wiener-Chaos decomposition $L_2(\Omega, \mathcal{F}, P) = \oplus_{n=0}^\infty H_n$. Any random variable of H_n can be expressed as a multiple Ito integral $I_n(f_n)$ of some symmetric kernel $f_n \in L_2([0, 1]^n; R^{dn})$.

Theorem 2.1 *(see [3]). Let be a square integrable random variable having an orthogonal Wiener-Chaos expansion of the form* $F = \sum_{n=0}^\infty I_n(f_n)$. *Then* F *belongs to the space* $D_{2,1}^w$ *if and only if* $\sum_{n=1}^\infty nn! \|f_n\|_{L_2([0,1]^n)} < \infty$ *and in this case we have* $D_t^w F = \sum_{n=1}^\infty n I_{n-1}(f_n(\cdot, t)), \ \ t \in [0, 1]$ *and*

$$\| \|D_{\cdot}^w F\|_{L_2([0,1])} \|_{L_2(\Omega)} = \sum_{n=1}^{\infty} nn! \|f_n\|^2_{L_2([0,1]^n)}.$$

Theorem 2.2 *(see [3]). Suppose* F *and* G *are smooth functionals and let* h *be an element of the Hilbert space* $L_2([0, 1]$. *Then we have*

$$E[G\langle D_{\cdot}^w F, h_{\cdot}\rangle_{L_2([0,1]}] = -E[F\langle D_{\cdot}^w G, h_{\cdot}\rangle_{L_2([0,1]} + FGw(h)].$$

III. Let $\sum_n$ be an increasing simplex of $R_+^n : \sum_n = \{(t_1, ..., t_n) \in R_+^n : 0 < t_1 < \cdots < t_n\}$, and extend a function f defined on $\sum_n$ by making symmetric on R_+^n. One can then define the multiple integral with respect to M as

$$I_n(f) := n! \int_{\sum_n} f(t_1, ..., t_n) dM_{t_1} \cdots dM_{t_n}.$$

Definition 3.1 (cf. Definition 3.2 [5]). Let $\mathcal{R} = \sigma\{M_t, t \geq 0\}$ be the σ-algebra generated by a normal martingale M. Let H_n be the n-th homogeneous chaos, $H_n = I_n(f)$, where f ranges over all $L_2(\sum_n)$. If

$L_2(\mathcal{R}, P) = \oplus_{n=0}^{\infty} H_n$, then we say that M possesses the chaos representation property (CRP).

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a filtered probability space satisfying the usual conditions. We assume that a normal martingale M with the CRP is given on it and that $\mathcal{F}$ is generated by M. Thus, for any random variable $F \in L_2(\mathcal{R}, P)$ we have by the CRP that there exists a sequence of symmetric functions $f_n \in L_2^s([0,1]^n)$, $n = 1, 2, ...$, such that $F = \sum_{n=0}^{\infty} I_n(f_n)$. Consider the following subset $D_{2,1}^M \subset L_2(\mathcal{R}, P)$

$$D_{2,1}^M = \{F = \sum_{n=0}^{\infty} I_n(f_n) : \sum_{n=1}^{\infty} nn! \|f\|^2_{L_2([0,1]^n)} < \infty\}.$$

Definition 3.2 (see [5]). The derivative operator is defined as a linear operator $D_{\cdot}^M$ from $D_{2,1}^M$ into $L_2([0,T] \times \Omega)$ by the relation:

$$D_t^M F := \sum_{n=1}^{\infty} n I_{n-1}(f_n(\cdot, t)), \quad t \in [0,1].$$

Example 3.1 (see [5]). Consider a symmetric function $f(s,t) = I(a,b](s) \times I(a,b](t)$. Using the Ito's formula the second chaos $I_2(f)$ can be computed as

$$\begin{aligned} I_2(f) = 2! \textstyle\int_{0<s<t\leq 1} f(s,t) dM_s dM_t = 2 \int_a^b \int_a^{t-} dM_s dM_t \\ = 2 \textstyle\int_a^b (M_{t-} - M_a) dM_t = (M_b - M_a)^2 - \{[M,M]_b - [M,M]_a\}. \end{aligned} \qquad (1)$$

Consider now the function $g(x,y) = (y-x)^2$, and define a smooth functional $F = g(M_a, M_b)$. Let us define the derivative $D_t^M F$ in a way analogous to one of the equivalent definitions in the Wiener case:

$$\begin{aligned} D_t^M F = D_t^M (M_b - M_a)^2 = \tfrac{\partial F}{\partial x}(M_a, M_b) I_{[0,a]}(t) + \tfrac{\partial F}{\partial y}(M_a, M_b) I_{[0,b]}(t) \\ = -2(M_b - M_a) I_{[0,a]}(t) + 2(M_b - M_a) I_{[0,b]}(t) = 2(M_b - M_a) I_{(a,b]}(t). \end{aligned} \qquad (2)$$

However, by the Definition 3.2, we have

$$D_t^M I_2(f) = 2I_1(f(\cdot, t)) = 2 \int_0^1 I_{(a,b]}(s) dM_s \cdot I_{(a,b]}(t) = 2(M_b - M_a) I_{(a,b]}(t).$$

We can substitute this into (2) and compare it with (1) to see that the two definitions coincide if and only if $D_t^M \{[M,M]_b - [M,M]_a\} = 0$, for all $t \in [0,1]$. By Lemma 4.1 [5], this means that $[M,M]_b - [M,M]_a$ must be constant. If we look at the structure equation (see (2.1) [5]), this amounts to saying that the two definitions are in contradiction and cannot hold simultaneously unless $M = w$, Wiener process.

IV. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}), P$ be a filtered probability space satisfying the usual conditions. Let N_t be the standard Poisson process and $\mathcal{F}_t$ is generated by $N, \mathcal{F} = \mathcal{F}_T$. Let M_t be the compensated Poisson process ($M_t = N_t - t$). Let us denote $\nabla_x f(x) := f(x+1) - f(x); \nabla_x f(M_T) := \nabla_x f(x)|_{x=M_T}$.

Using the relations $M_s = \int_0^T I_{[0,s]}(u)dM_u = I_1(I_{[0,s]}(\cdot))$ and $[M,M]_s = N_s = M_s + s$, by the Definition 3.2 we can obtain:

Proposition 4.1. $D_t^M M_s = D_t^M[I_1(I_{[0,s]}(\cdot))] = I_{[0,s]}(t) and$

$$D_t^M[M,M]_s = D_t^M N_s = D_t^M M_s + D_t^M s = I_{[0,s]}(t).$$

Definition 4.1. a). $\overline{D}_t^M (M_s)^n := [\nabla_x(x^n)]|_{x=M_s} \cdot \overline{D}_t^M M_s := [\nabla_x(x^n)]|_{x=M_s} \times I_{[0,s]}(t)$; **b).** For any polynomial function $P_m(x_1, ...x_n)$:

$$\overline{D}_t^M P_m(M_{t_1}, ..., M_{t_n}) := \sum_{k=1}^{n} \sum_{i_1<i_2<...<i_k} \nabla_{x_{i_1}} \cdots \nabla_{x_{i_k}} P_m(M_{t_1}, ..., M_{t_n})$$
$$\times I_{[0,t_{i_1}]}(t) \cdots I_{[0,t_{i_k}]}(t).$$

Remark 4.1. If we take here $n = 2$, we obtain that

$$\overline{D}_t^M M_s^2 = \nabla_x x^2|_{x=M_s} \cdot \overline{D}_t^M M_s = (2M_s + 1)I_{[0,s]}(t),$$

whereas in the Wiener process ases

$$D_t^w w_s^2 = \frac{\partial}{\partial x} x^2|_{x=w_s} \cdot D_t^w w_s = 2w_s I_{[0,s]}(t).$$

Theorem 4.1. *If $F = I_n(f_n)$ for some $f_n \in L_2^s([0,T]^n)$, then $\overline{D}_t^M F = nI_{n-1}(f_n(\cdot,t))$ and $\|\overline{D}_t^M F\|^2_{L_2([0,T]\times\Omega)} = nn!\|f_n\|^2_{L_2([0,T]^n)}$.*

Proof. Let $f_n(t_1, ..., t_n) = \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n} I_{A_{i_1}\times\cdots A_{i_n}}(t_1, ..., t_n)$, then we have

$$I_n(f_n) = \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n} \int_0^T I_{A_{i_1}}(s)dM_s \cdots \int_0^T I_{A_{i_n}}(s)dM_s :$$
$$= \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n} M(A_{i_1}) \cdots M(A_{i_n}).$$

Therefore, due to the Definition 4.1, one can easily verify that:

$$\overline{D}_t^M I_n(f_n) = \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n} \overline{D}_t^M [M(A_{i_1}) \cdots M(A_{i_n})]$$
$$= \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n} I_{A_{i_1}}(t) \cdots I_{A_{i_n}}(t) + \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n}$$
$$\times \sum_{j=1}^n I_{A_{i_1}}(t) \cdots I_{A_{i_{j-1}}}(t) I_{A_{i_{j+1}}}(t) \cdots I_{A_{i_n}}(t) M(A_{i_j}) + \cdots$$
$$+ \sum_{i_1,...,i_n=1}^m a_{i_1,...,i_n} \sum_{j=1}^n I_{A_{i_j}}(t) M(A_{i_1}) \cdots M(A_{i_{j-1}}) M(A_{i_{j+1}}) \cdots M(A_{i_n})$$
$$= nI_{n-1}(f_n(\cdot, t).$$

Moreover, it is not difficult to see that:

$$\|\overline{D}_t^M F\|^2_{L_2([0,T]\times\Omega)} = \int_0^T \|nI_{n-1}(f_n(\cdot,t))\|^2_{L_2(\Omega)} dt$$
$$= nn! \int_0^T \|f_n(\cdot,t)\|^2_{L_2([0,T]^{n-1})} dt = nn!\|f_n\|^2_{L_2([0,T]^n)}.$$

Analogously one can prove the following

Theorem 4.2. *For Poisson polynomial functionals the above-given two definitions of stochastic derivatives (Definition 3. 2 from [4] and Definition 4.1) are equivalent:* $D_t^M P_m(M_T) = \overline{D}_t^M P_m(M_T)$.

Remark 4.2. It is not difficult to see that the Definition 4.1 is agree with the Example 3.1. Indeed, due to the Proposition 4.1 and Theorem 4.1, we have

$$\overline{D}_t^M I_2\{I_{(a,b]}(\cdot)I_{(a,b]}(\cdot)\} + \overline{D}_t^M\{[M,M]_b - [M,M]_a\}$$
$$= 2I_1\{I_{(a,b]}(\cdot)\}I_{(a,b]}(t) + I_{[0,b]}(t) - I_{[0,a]}(t)$$
$$= [2(M_b - M_a) + 1] \cdot [I_{[0,b]}(t) - I_{[0,a]}(t)].$$

On the other hand, using the Definition 4.1, we can write

$$\overline{D}_t^M (M_b - M_a)^2 = \overline{D}_t^M M_b^2 - 2\overline{D}_t^M (M_b M_a) + \overline{D}_t^M M_a^2$$
$$= (2M_b + 1)I_{[0,b]}(t) - 2[M_a I_{[0,b]}(t) + M_b I_{[0,a]}(t) + I_{[0,a]}(t)I_{[0,b]}(t)]$$
$$+(2M_a + 1)I_{[0,a]}(t) = [2(M_b - M_a) + 1] \cdot [I_{[0,b]}(t) - I_{[0,a]}(t)].$$

Proposition 4.2. *Let* F *be a random variable of the space* $D_{2,1}^M$ *such that* $\overline{D}_t^M F = 0$ *for all* $t \in [0,T]$*. Then* $F = EF$*.*

Proof. Suppose that $F = \sum_{n=0}^{\infty} I_n(f_n) = f_0 + \sum_{n=1}^{\infty} I_n(f_n)$. Taking the mathematical expectation from the both sides of the last relation we obtain that: $f_0 = EF$. On the other hand, since $\overline{D}_t^M F = 0$ for all $t \in [0,T]$, we can write:

$$0 = E\int_0^T \overline{D}_t^M F dt = \sum_{n=1}^{\infty} nn!\|f_n\|^2_{L_2([0,T]^n)}.$$

From here we conclude that $f_0 = 0$ a.s. for all $n \geq 1$, and hence, $I_n(f_n) = 0$ for all $n \geq 1$. Therefore, $F = f_0 = EF$.

Let $A \in \mathcal{B}([0,T])$. We will denote by $\mathcal{F}_A$ the σ-algebra (completed with respect to the probability P) generated by the random variables

$$M(B) = \int_B dM_t \ \ (B \subset A, \ B \in \mathcal{B}([0,T])).$$

Proposition 4.3. *Suppose that* $F = \sum_{n=0}^{\infty} I_n(f_n)$ *and* $A \in \mathcal{B}([0,T])$*. Then we have:*

$$E(F|\mathcal{F}_A) = \sum_{n=0}^{\infty} I_n(f_n I_{A^n}) := \sum_{n=0}^{\infty} I_n(f_n I_A^{\otimes n}).$$

Proof. It suffices to assume that $F = I_n(f_n)$, where f_n is a symmetric and elementary kernel. Also, by linearity we can assume that the kernel is of the form: $f_n = I_{B_1\times\cdots\times B_n}$, where $B_1, ..., B_n$ are mutually disjoint sets of finite measure. In this case we have:

$$E(F|\mathcal{F}_A) = E[M(B_1)\cdots M(B_n)|\mathcal{F}_A] = E\{[M(B_1\cap A) + M(B_1\cap \overline{A})]$$

$$\cdots[M(B_n\cap A) + M(B_n\cap \overline{A})]|\mathcal{F}_A\} = M(B_1\cap A)\cdots M(B_n\cap A)$$

$$= \textstyle\sum_{n=0}^{\infty} I_n(f_n I_A^{\otimes n}).$$

Proposition 4.4. *Suppose that F belongs to the space $D_{2,1}^M$, and let $A\in\mathcal{B}([0,T])$. Then $E(F|\mathcal{F}_A)$ also belongs to $D_{2,1}^M$ and we have:*

$$\overline{D}_t^M E(F|\mathcal{F}_A) = E(\overline{D}_t^M F|\mathcal{F}_A)I_A(t) \ \text{ a.s. in } [0,T]\times\Omega.$$

Proof. Due to the Proposition 4.3 and Definition 3.2, on the one hand, we can write:

$$\overline{D}_t^M E(F|\mathcal{F}_A) = \sum_{n=1}^{\infty} nI_{n-1}[f_n(\cdot,t)I_A^{\otimes(n-1)}]I_A(t).$$

On the other hand, we have:

$$E(\overline{D}_t^M F|\mathcal{F}_A) = E\{\sum_{n=1}^{\infty} nI_{n-1}[f_n(\cdot,t)]|\mathcal{F}_A\} = \sum_{n=1}^{\infty} nI_{n-1}[f_n(\cdot,t)I_A^{\otimes(n-1)}].$$

Corollary 4.1. Suppose that $F\in D_{2,1}^M$ and F is $\mathcal{F}_t^M$-measurable. Then $\overline{D}_s^M F = 0$ for all $s > t$.

Denote by $\langle\cdot,\cdot\rangle$ the scalar product in $L_2([0,T])$.

Theorem 4.3. *Let h be an element of the Hilbert space $L_2([0,T])$. Then for any polynomial functions $f(x)$ and $g(x)$ we have*

$$E[g(M_T)\langle\overline{D}_\cdot^M f(M_T), h_\cdot\rangle] = -E[f(M_T+1)\langle\overline{D}_\cdot^M g(M_T), h_\cdot\rangle]$$

$$+E[f(M_T)g(M_T)M(h)]/T.$$

Proof. Using the distribution of N_T, it is not difficult to verify that the following relations are valid:

$$E[g(M_T)\langle\overline{D}_\cdot^M f(M_T), h_\cdot\rangle] = E[g(M_T)\langle\nabla_x f(M_T)I_{[0,T]}(\cdot), h_\cdot\rangle]$$

$$= \langle I_{[0,T]}(\cdot), h_\cdot\rangle \textstyle\sum_{x=0}^{\infty} g(x-T)\nabla_x f(x-T)T^x e^{-T}/x!$$

$$= [\langle I_{[0,T]}(\cdot), h_\cdot\rangle g(x-T)\nabla_x f(x-T)T^x e^{-T}/x!]|_0^{\infty}$$

$$-\langle I_{[0,T]}(\cdot), h_\cdot\rangle \textstyle\sum_{x=0}^{\infty} f(x+1-T)\nabla_x[g(x-T)T^x e^{-T}/x!]$$

$$= -\langle I_{[0,T]}(\cdot), h_\cdot\rangle\{g(-T)f(-T)e^{-T} - \textstyle\sum_{x=0}^{\infty} f(x+1-T)g(x+1-T)$$

$$\times\frac{(x+1-T)T^{x+1}}{T(x+1)!}e^{-T} + E[f(M_T+1)\nabla_x g(M_T)]\}$$

$$= -\langle I_{[0,T]}(\cdot), h_\cdot\rangle\{g(-T)f(-T)e^{-T} - E[f(M_T)g(M_T)M_T/T]$$

$$-f(-T)g(-T)e^{-T}\} - E[f(M_T+1)\langle\nabla_x g(M_T)I_{[0,T]}(\cdot), h_\cdot\rangle]$$

$$= E[f(M_T)g(M_T)M(h)]/T - E[f(M_T+1)\langle\overline{D}_\cdot^M g(M_T), h_\cdot\rangle].$$

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REFERENCES

1. Skorokhod A.V. On a generalization of a stochastic integral, *Theor. Prob. Appl.,* **20** (1975), 219-233.

2. Gaveau B., Trauber P. L'integrale stochastique comme operateur de divergence dans l'espace fonctionnel, *J. Funct. Anal.*, **46**, (1982), 230-238.

3. Ocone D. Malliavin's calculus and stochastic integral representation of functionals of diffusion processes, *Stochastics,* **12** (1984), 161-185.

4. Nualart D., Pardoux E. Stochastic calculus with anticipating integrands, *Probab. Theor. Rel. Fields* **78**, 1988, 535-581.

5. Ma J., Protter P., Martin J.S. Anticipating integrals for a class of martingales, *Bernoulli*, **4**, 1 (1998), 81-114.

6. Harrison J. M., Pliska S.R. Martingales and stochastic integrals in the theory of continuous trading, *Stochastic Process, Appl.,* **11** (1981), 215-260.

7. Jaoshvili V., Purtukhia O. An extension of the Ocone-Haussmann-Clark formula for the compensated Poisson process, *Th. Probab. Appl.,* **53**, 2 (2008), 349-354.

8. Purtukhia O. An extension of the Ocone-Haussmann-Clark formula for a class of normal martingales, *Proc A. Razmadze Math. Inst.,* **132**, (2003), 127-136.

9. Jaoshvili V., Purtukhia O. Stochastic integral representation of functionals of Poisson processes, *Proc. A. Razmadze Math. Inst.,* **143** (2007), 37-60.

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