

**ПРОБЛЕМЫ СОВРЕМЕННОЙ
МАТЕМАТИКИ**

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**22-23 АПРЕЛЯ, 2011 ГОДА
КАРШИ**

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ТРУДЫ

**НАУЧНОЙ КОНФЕРЕНЦИИ
ПРОБЛЕМЫ СОВРЕМЕННОЙ
МАТЕМАТИКИ**

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On the modeling of the standard options pricing process

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Abstract. The problem of the pricing of European and American type standard options is investigated for the Cox-Ross-Rubinstein discrete model of financial market. The form of a fair price and minimal hedge is found for one class of nonselffinanced strategies. The obtained results make it possible to construct a complex of programs for numerical examples.

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1. We consider the financial (B, S) – market consisting only of two assets: a bonds – $B = (B_n)$ and stocks – $S = (S_n)$, $n = 0, 1, \dots, N$. According to the Cox-Ross-Rubinstein model we have

$$B_n = (1+r)B_{n-1}, \quad B_0 > 0, \quad (1)$$

$$S_n = (1+\rho_n)S_{n-1}, \quad S_0 > 0, \quad (2)$$

where $r > 0$ is an interest rate, ρ_n is a sequence of independent, identically distributed random variables taking only two values a and b , $a < r < b$ (see [1]).

Let us now assume that there is some investor who has the initial capital $X_0 = x > 0$ and wants to raise this capital in the future by using the capability of the (B, S) – market. In that case, we deal with the so-called investment problem. Suppose that at the moment $n=0$ the investor purchased β_0 quantity of bonds and γ_0 quantity of stocks. Then we have

$$X_0 = X_0^\pi = \beta_0 B_0 + \gamma_0 S_0, \quad (3)$$

where $\pi = \pi_0 = (\beta_0, \gamma_0)$ is the investor's portfolio or strategy .

Assume that there is a sequence of functions $g = (g_n)$, $n = 0, 1, \dots, N$, $g_0 = 0$, and the investor transformed his portfolio π_0 to the new portfolio $\pi_1 = (\beta_1, \gamma_1)$ in a maner such that the equality

$$X_0^\pi = \beta_1 B_0 + \gamma_1 S_0 + g_1 \quad (4)$$

is satisfied. At the moment $n = 1$ we have

$$X_1^\pi = \beta_1 B_1 + \gamma_1 S_1. \quad (5)$$

Analogously, for any moments $n-1$ and n we have

$$X_{n-1}^\pi = \beta_n B_{n-1} + \gamma_n S_{n-1} + g_n, \quad (6)$$

$$X_n^\pi = \beta_n B_n + \gamma_n S_n. \quad (7)$$

The strategy $\pi_n = (\beta_n, \gamma_n)$ is called nonselffinanced. If $g_n \equiv 0$ then π_n is called selffinanced. If $X_0^\pi = X_0 = x$ and $X_N^\pi \geq f_N$ then π_n is called a hedge, where $f = f_N$ is some payoff function, if $X_N^\pi = f_N$ then π_n is called a minimal hedge. Denote by Π the set of all hedges.

2. Now let us define a standard European call option. This is a derivative security with the payoff function

$$f = f_N = \max(S_N - K, 0). \quad (8)$$

The owner of this option enjoys the right to buy a stock at a price K at a certain moment N . If $S_N > K$, then the owner will buy a stock at a price K , sell it at a price S_N and have a gain

$$f_N = S_N - K - C_N, \quad (9)$$

where $C_N = \min\{x > 0 : \Pi = \emptyset\}$ is the so called fair price of option.
The problem of the investor (option seller) consist in the following: using the fair price C_N it is required to construct a minimal hedge.

Assume that the sequence of functions $g = (g_n)$ defined by the equality

$$g_n = c_1 \beta_n B_{n-1} + c_2 \gamma_n S_{n-1}, \quad 0 < c_1 < 1, \quad 0 < c_2 < 1. \quad (10)$$

Lemma 1. At each moment n , $n = 0, 1, \dots, N-1$, a minimal hedge $\pi_{n+1}^* = (\beta_{n+1}^*, \gamma_{n+1}^*)$ is defined by the following equalities

$$\beta_{n+1}^* = \frac{(1+b)f((1+a)S_n) - (1+a)f((1+b)S_n)}{(1+r)(b-a)B_n}, \quad (11)$$

$$\gamma_{n+1}^* = \frac{f((1+a)S_n) - f((1+b)S_n)}{(b-a)B_n}, \quad (12)$$

where f is some payoff function.

Lemma 2. The capital of the minimal hedge π_{n+1}^* is defined by the equality

$$X_n^{\pi^*} = \frac{1+c_1}{1+r} \cdot [pf((1+b)S_n) + (1-p)f((1+a)S_n)], \quad (13)$$

where

$$p = \frac{r - c_1(1+a) + c_2(1+r) - a}{(b-a)(1+c_1)}. \quad (14)$$

Theorem 1. The fair price of an European standard call option is defined by the following equality

$$C_N = S_0 \sum_{k=k_0}^N C_N^k p^k (1-p)^{N-k} \left(\frac{(1+c_1)(1+a)}{1+r} \right)^N \left(\frac{1+b}{1+a} \right)^k - K \cdot \left(\frac{1+c_1}{1+r} \right)^N \sum_{k=k_0}^N C_N^k \cdot p^k \cdot (1-p)^{N-k}, \quad (15)$$

where k_0 is the smallest integer number for which the inequality

$$S_0 \cdot ((1+c_1)(1+a))^N \cdot \left(\frac{1+b}{1+a} \right)^{k_0} > K.$$

Suppose that

$$B_0 = 20, \quad r = \frac{1}{5}, \quad S_0 = 100, \quad K = 100, \\ \rho_n = b = \frac{3}{5} \quad \text{or} \quad \rho_n = a = -\frac{2}{5}, \quad n = 0, 1, \dots, N. \quad (16)$$

Example 1. Let we have the function (8), $N = 2$ ($n = 0, 1, 2$), $c_1 = \frac{1}{40}$, $c_2 = \frac{1}{50}$. Then we have

$$C_2 = \frac{609 \cdot 203 \cdot 13}{40000}, \quad \pi_1^* = \left(-\frac{609 \cdot 13}{4000}, \frac{609 \cdot 13}{10000} \right), \\ g_1 = \frac{609 \cdot 13 \cdot 3}{40000}, \quad X_0^{\pi^*} = C_2.$$

Case I. $S_1 = 160$, $\pi_2^* = \left(\frac{13}{40}, \frac{39}{40}\right)$, $g_2 = \frac{117}{100}$.

1) If $S_2 = 256$, then $X_2^{\pi^*} = f(S_2) = 156$,

2) If $S_2 = 96$, then $X_2^{\pi^*} = f(S_2) = 0$.

Case II. $S_1 = 60$, $\pi_2^* = (0, 0)$, $g_2 = 0$.

1) If $S_2 = 96$, then $X_2^{\pi^*} = f(S_2) = 0$,

2) If $S_2 = 36$, then $X_2^{\pi^*} = f(S_2) = 0$.

3. Consider now the American standard put option with the payoff function $f = f_n = \max(K - S_n, 0)$. (17)

Let we have the sequence of functions (17) and introduce the operator

$$Tf(x) = \frac{1+c_1}{1+r} \cdot [pf((1+b)x) + (1-p)f((1+a)x)], \quad (18)$$

where p is defined by (14).

Lemma 3. The fair price of the American standard put option $P_n^A(x)$ satisfies the following recurrent equations

$$P_n^A(x) = \max(f_n(x), TP_{n+1}^A(x)), \quad n = 0, 1, \dots, N-1. \quad (19)$$

lemma 4. The rational (optimal stopping) moment τ^* is defined by the inequality

$$f(S_n) \geq TP_{n+1}^A(S_n). \quad (20)$$

Let's introduce the following notations

$$S_{1,j} = S_0 (1+b)^j (1+a)^{1-j},$$

$$f_{1,j} = f(S_{1,j}), \quad j = 0, 1; \quad N = 1, \quad n = 0, 1;$$

$$S_{2,j} = S_0 (1+b)^j (1+a)^{2-j},$$

$$f_{2,j} = f(S_{2,j}), \quad j = 0, 1, 2; \quad N = 2, \quad n = 0, 1, 2.$$

Theorem 2. The fair (rational) price of the American standard put option can be calculated by the following recurrent equality

$$P_{N-k,j}^A = \max \left\{ f_{N-k,j}, \frac{1+c_1}{1+r} \cdot [pP_{N-k+1,j+1}^A + (1+p)P_{N-k+1,j}^A] \right\}, \quad (21)$$

where p is defined by (14), $k = 0, 1, \dots, N$; $j = 0, 1, \dots, N-k$.

Example 2. Let we have the values (16) and the function (17). Suppose that $c_1 = c_2 = 0$, $N = 2$ ($n = 0, 1, 2$). Then we have

$$P_2^A = P_{0,0}^A = \frac{42}{3}, \quad \pi_1^* = \left(\frac{79}{30}, -\frac{29}{25}\right), \quad X_0^{\pi^*} = P_2^A.$$

1) If $S_{1,1} = 160$, then $\pi_2^* = \left(\frac{2}{9}, -\frac{1}{40}\right)$, $\tau^* = 2$,

2) If $S_{1,0} = 60$, then $\pi_2^* = \left(\frac{125}{36}, -1\right)$, $\tau^* = 1$.

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Some results for the first-order autoregressive model

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We shall consider the first-order autoregressive process (Y_1, Y_2, \dots) defined by

$$Y_t = \beta Y_{t-1} + u_t, \quad t = 2, 3, \dots, \quad (1)$$

where $|\beta| < 1$, $\{u_1, u_2, \dots\}$ is a sequence of independent and identically distributed (i.i.d.) $N(0, \sigma^2)$ random variables. If $\bar{\beta}$ denotes the LS estimator of β based on n observations (Y_1, Y_2, \dots, Y_n) , then the s -periods-ahead LS forecast is $\bar{Y}_{n+s} = \bar{\beta}^s Y_n$. The least-squares estimator of β is:

$$\bar{\beta} = \frac{\sum_{t=2}^n y_t y_{t-1}}{\sum_{t=2}^n y_{t-1}^2}, \quad (2)$$

In the non-stationary case the least-squares estimator is the maximum likelihood estimator. The forecast error is therefore

$$\bar{Y}_{n+s} - Y_{n+s} = (\bar{\beta}^s - \beta^s) Y_n - \sum_{j=0}^{s-1} \beta^j u_{n+s-j} \quad (3)$$

From (3) we obtain the forecast bias,

$$M(\bar{Y}_{n+s} - Y_{n+s}) = M(\bar{\beta}^s Y_n), \quad (4)$$

and the mean-square forecast error (MSFE),

$$M(\bar{Y}_{n+s} - Y_{n+s})^2 = \beta^{2s} \text{var}(Y_n) + M(\bar{\beta}^{2s} Y_n^2) - 2\beta^s M(\bar{\beta}^s Y_n^2) + \sigma^2 \sum_{j=0}^{s-1} \beta^{2j}. \quad (5)$$

Some properties estimator (2) was considered in [1]. From (4) we see that the expectation of the forecast error exist if and only if, $M(\bar{\beta}^s Y_n)$ exists, that is, if and only if $0 \leq s < n-1$. Similarly, from (5), we see that the MSFE exists if and only if $M(\bar{\beta}^{2s} Y_n^2)$, $M(\bar{\beta}^s Y_n^2)$ exist, that is, if and only if, $0 \leq s < (n-1)/2$

Theorem. The expectation of the forecast error of \bar{Y}_{n+s} exists if and only if $1 \leq s \leq n-2$, in which case

$$M(\bar{Y}_{n+s} - Y_{n+s}) = 0,$$

since $\bar{\beta}$ is a consistent estimator of β

$$\lim_{n \rightarrow \infty} MSFE = \sigma^2 (1 - \beta^{2s}) / (1 - \beta^2)$$

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