Reconsideration of the 2-Flavor NJL Model with Dimensional Regularization at Finite Temperature and Density

Takahiro Fujihara,[1] Tomohiro Inagaki,[2] Daiji Kimura[1] and Alexander Kvinikhidze[3]

[1] *Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan*
[2] *Information Media Center, Hiroshima University, Higashi-Hiroshima 739-8521, Japan*
[3] *A. Razmadze Mathematical Institute of Georgian Academy of Sciences, M. Alexidze Str. 1, 380093 Tbilisi, Georgia*

We study the 2-flavor Nambu–Jona-Lasinio (NJL) model at finite temperature, T, and chemical potential, μ, in the dimensional regularization. The pion and sigma meson masses are calculated at finite T and μ. The color superconductivity is also discussed in the extended NJL model. We obtain the phase structure of the chiral and color symmetry in the dimensional regularization and compare the results with ones obtained in the cutoff regularization.

§1. Introduction

The asymptotic freedom of QCD and consequently perturbation theory unfortunately is of no help in the study of the low energy phenomena of QCD; therefore effective models of QCD are widely used. Here we regard the NJL model as a low energy effective model of QCD and evaluate the phase diagram at finite temperature and chemical potential. NJL model contains a four-fermion interaction corresponding to a dimension 6 operator in the Lagrangian. Thus the model is non-normalizable in four space-time dimensions and some regularization should be used. It is not surprising that the results may depend on the regularization scheme. One usually introduces the cutoff scale to regularize the model. The scale is fixed to reproduce the pion physics. Unfortunately the Fermi momentum exceeds the cutoff parameter in the color superconducting phase.

Schwinger-Dyson (SD) equation is a commonly used means to consider nonperturbative effects in QCD. One can evaluate the dynamically generated fermion (quark) mass by using SD equation for fermion propagator. It is known that SD equation gives similar results to NJL model in two dimensions. If we apply the ladder approximation, the instantaneous exchange approximation and neglect momentum dependent parts of the fermion self-energy in SD equation, the result coincides with the gap equation in two dimensional NJL model at the leading order of $1/N_c$ expansion. Two dimensional NJL model is renormalizable, but too simple for our proposes; therefore here we consider the model in D $(2 < D < 4)$ dimensions.

§2. Meson masses

The two-flavor NJL model is defined by[1),2)]

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\}, \tag{2·1}$$

where m is the quark mass matrix, $m = {\rm diag}(m_u, m_d)$, g is an effective coupling constant, and $\vec{\tau}$ represents the isospin Pauli matrices.

We investigate the pion and sigma meson masses at finite temperature and chemical potential in the dimensional regularization. To find the meson mass we calculate its propagator via summation of the bubble type (fermion loop) diagrams. The propagator for the scalar, G_s, and pseudo-scalar channel, G_5, in the leading order of the $1/N_c$ expansion is given by

$$G_{s,5}(p^2, \langle\sigma\rangle) = \frac{4g^2}{2g - \Pi_{s,5}(p^2)}, \tag{2·2}$$

where the self-energy $\Pi_{s,5}(p^2)$ is

$$\Pi_{s,5}(p^2) = 4ig^2 \int \frac{d^D k}{(2\pi)^D} {\rm tr}\left[\Gamma_{s,5}\frac{1}{k\!\!\!/ - m - \langle\sigma\rangle}\Gamma_{s,5}\frac{1}{(k\!\!\!/ - p\!\!\!/) - m - \langle\sigma\rangle}\right], \tag{2·3}$$

with $\Gamma_{s,5} = (\Gamma_s, \Gamma_5) = (1, i\gamma_5)$. We regularize the self-energy (2·3) by taking the space-time dimensions as a parameter ($2 < D < 4$). After having renormalized the coupling constant,[3)] we calculate the pion and sigma meson masses as the solutions of the equations

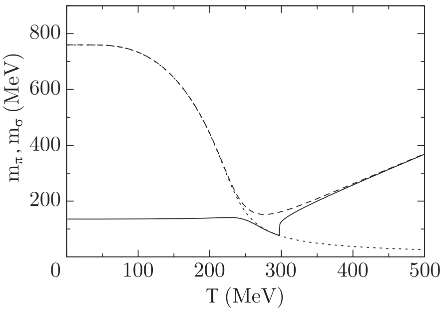
$$G_5^{-1}(p^2 = m_\pi^2, \langle\sigma\rangle) = 0, \quad {\rm Re}[G_s^{-1}(p^2 = m_\sigma^2, \langle\sigma\rangle)] = 0, \tag{2·4}$$

using the gap equation. These are not applicable in some cases, for example, high temperature and/or large chemical potential. In such cases we use

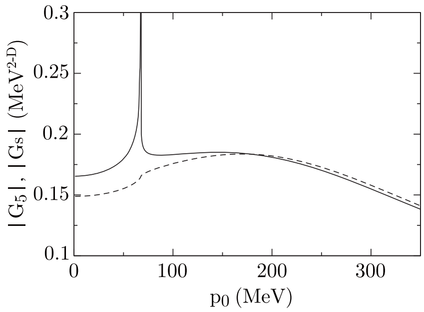
$$\frac{\partial}{\partial p}|G_{s,5}(p^2 = m_{\sigma,\pi}^2, \langle\sigma\rangle)| = 0. \tag{2·5}$$

We use the imaginary time formalism, to introduce the temperature and the chemical potential. At the finite T and μ, the denominator of the fermion propagator in Eq. (2·3) changes from $k\!\!\!/ - m - \langle\sigma\rangle$ to $k_i\gamma_i - (\omega_n - i\mu)\gamma_4 + m + \langle\sigma\rangle$, $\omega_n = (2n+1)\pi T$. The pion and sigma meson masses are obtained by solving Eq. (2·4) or (2·5). To evaluate the meson masses at finite T and μ, we fix the value of the model parameters g, D [*)] and renormalization scale by calculating the pion mass and decay constant with $m_u = m_d = 5$ MeV in the dimensional regularization at $T = \mu = 0$.[3)] In Fig. 1(a) we plot the pion and sigma meson masses as functions of the temperature at $\mu = 100$ MeV. The soft mode of the pion and sigma meson channels almost degenerate above $T \simeq 300$ MeV. We also draw the behavior of $|G_5|$ and $|G_s|$ at $\mu = 100$ MeV and $T = 310$ MeV in Fig. 1(b). The sharp peak structure of pion channel corresponds to the term $-2(\langle\sigma\rangle + m)$ which is shown by the dotted line in Fig. 1(a).

[*)] One degree of freedom still remains. We choose $D = 2.4$ to evaluate the meson masses.



(a) Pion and sigma meson masses.



(b) $|G_5|$ and $|G_s|$ at $T = 310$ MeV.

Fig. 1. (a) Pion and sigma meson masses as functions of the temperature at $\mu = 100$ MeV. The full line represents the pion channel, the dashed line the sigma channel, and the dotted line $-2(\langle\sigma\rangle + m)$. (b) Meson propagator as functions of p_0 at $\mu = 100$ MeV. The full line represents the pion propagator $|G_5|$, the dashed line the sigma meson propagator $|G_s|$.

§3. Phase structure

At some large chemical potential a color Cooper pair of quarks can be created in the $\bar{3}$ irreducible representation of the $SU(3)$ color symmetry, i.e. color superconductivity phase transition takes place. To study the color superconducting phase the NJL model is extended to include the extra interaction term which is written in the diquark channel for the convenience. The model is given by

$$\mathcal{L} = \mathcal{L}_{\rm NJL} + g_D(\bar{\psi}^C_{ai}\varepsilon_{ij}\epsilon^b_{ac}i\gamma_5\psi_{cj})(\bar{\psi}_{dk}\varepsilon_{kl}\epsilon^b_{de}i\gamma_5\psi^C_{el}), \qquad (3\cdot1)$$

where the indices $a, b, c, \cdots$ and $i, j, k, \cdots$ denote the color (1,2,3) and flavor (u, d). ψ^C represents the charge conjugate of ψ and g_D is the effective coupling constant for the diquark channel.

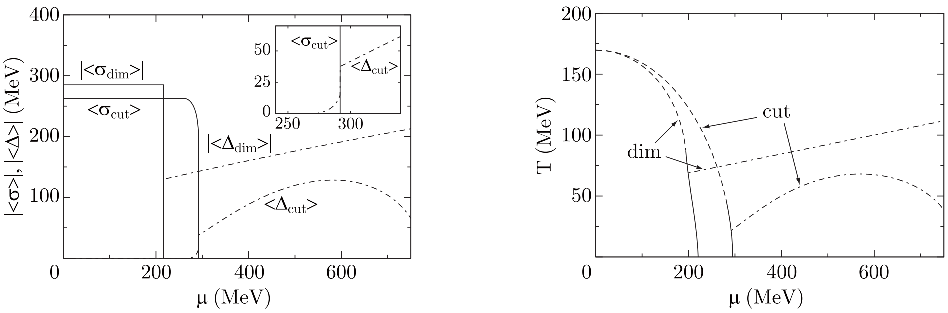
For practical calculations it is more convenient to introduce the auxiliary fields, scalar σ, pseudo-scalar $\vec{\pi}$ and diquark $\Delta^b \sim -G_D\bar{\psi}^C\varepsilon\epsilon^b i\gamma_5\psi$. For the simplicity, we take the massless quark limit and $\vec{\pi} = 0$ owing to the chiral $SU(2)$ symmetry. The color $SU(3)$ symmetry allows us to set $\Delta^{1,2} = 0$ and $\Delta^3 = \Delta$.

To examine the phase structure of the extended NJL model we evaluate the effective potential at finite T and μ.

$$V_{\rm eff}(\sigma, \Delta) = \frac{\sigma^2}{4g} + \frac{\Delta^2}{4g_D} - \tilde{A}\int_0^\infty dp\, p^{D-2}\Big[E_+ + E_- + E + 2T\ln(1+e^{-E_+/T}) + 2T\ln(1+e^{-E_-/T}) + T\ln(1+e^{-\xi_+/T}) + T\ln(1+e^{-\xi_-/T})\Big], \qquad (3\cdot2)$$

with $E \equiv \sqrt{p^2+\sigma^2}$, $\xi_\pm \equiv (E\pm\mu)$, $E_\pm \equiv \sqrt{\Delta^2+\xi_\pm^2}$ and $\tilde{A} \equiv 4\sqrt{\pi}/[(2\pi)^{D/2}\Gamma\left(\frac{D-1}{2}\right)]$. The difference of the effective potential (3·2) from one derived in the cutoff regularization ($D = 4$) is only in the third term of the right-hand side

$$\tilde{A}\int_0^\infty dp\, p^{D-2} \quad \rightarrow \quad \frac{2}{\pi^2}\int_0^\Lambda dp\, p^2, \qquad (3\cdot3)$$



(a) Solution of the gap equation.

(b) Phase diagram in T-μ plane.

Fig. 2. (a) Typical behavior of the gap equation's solution, $|\langle\sigma\rangle|$ and $|\langle\Delta\rangle|$ at $T \simeq 0$. The solid and dashed-dotted lines represent $|\langle\sigma\rangle|$ and $|\langle\Delta\rangle|$ respectively. (b) The dashed and solid lines indicate the second and first order phase transition of the chiral symmetry breaking, the dashed-dotted line the second order phase transition of the color symmetry breaking.

where Λ is a cutoff scale.

Using the effective potential (3·2) with Eq. (3·3), we obtain the solution of the gap equation in Fig. 2(a) and the phase structure of the chiral and color symmetry in Fig. 2(b).*) $\langle\Delta\rangle$ in the cutoff regularization decreases near the cutoff scale in Fig. 2(a). There is not coexistence phase of $\langle\sigma\rangle$ and $\langle\Delta\rangle$ in the dimensional regularization. Color superconducting phase in the cutoff regularization also goes down near the cutoff scale in Fig. 2(b). The tricritical point is located $(T,\ \mu) = (87.2,\ 194)$ and $(46.7,\ 281)$ for the dimensional and cutoff respectively.

§4. Summary

We have discussed the meson masses and the phase structure of the two-flavor NJL model at finite T and μ using the dimensional regularization. The behavior of the meson masses at small μ is similar to that of the cutoff theory. It is found that the symmetry breaking of the chiral and color symmetry become different as μ is increased. In this region one can see a new physical aspect in the NJL model using the dimensional regularization. A paper about the susceptibility and the high density stars is in preparation.

References

1) Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961), 345; Phys. Rev. **124** (1961), 246.
2) K. Higashijima, Prog. Theor. Phys. Suppl. No. 104 (1991), 1.
B. Rosenstein, B. J. Warr and S. H. Park, Phys. Rep. **205** (1991), 59.
U. Vogl, Prog. Part. Nucl. Phys. **27** (1991), 1995.
S. P. Klevansky, Rev. Mod. Phys. **64** (1992), 649.
T. Hatsuda and T. Kunihiro, Phys. Rep. **247** (1994), 221.
3) T. Inagaki, D. Kimura and A. Kvinikhidze, arXiv:0712.1336; to be published in Phys. Rev. D.

*) We fix the parameters in the cutoff($\Lambda = 720$ MeV) and dimensional regularizations to reproduce the pion mass and decay constant with $m_u = m_d = 4.5$ MeV. We set $D = 2.28$ in order to get the same order critical temperature at $\mu = 0$ as in the cutoff regularization.