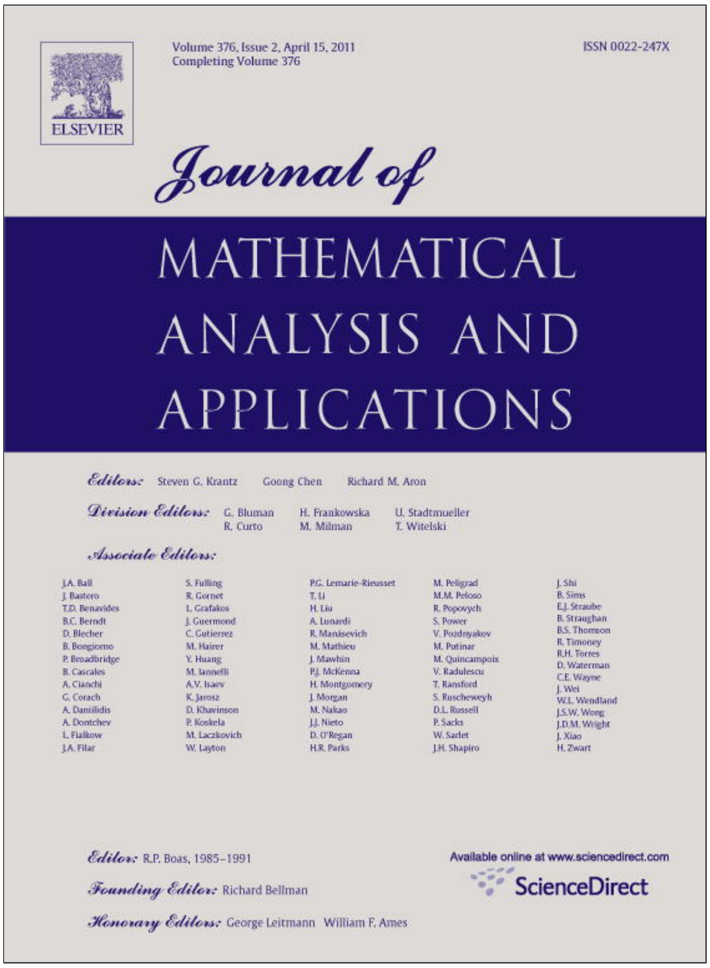
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Global solvability of the Cauchy characteristic problem for one class of nonlinear second order hyperbolic systems

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ABSTRACT

The Cauchy characteristic problem in the light cone of the future for one class of nonlinear hyperbolic systems of the second order is considered. The existence and uniqueness of global solution of this problem is proved.

1. Introduction

In the Euclidean space $\mathbb{R}^{n+1}$ of the independent variables $x_1, x_2, \dots, x_n, t$ consider a nonlinear hyperbolic system of the form

$$\Box u_i + \lambda \frac{\partial}{\partial u_i} G(u_1, \dots, u_N) = F_i(x,t), \quad i = 1, \dots, N, \tag{1.1}$$

where λ is a given real constant, G is a given real scalar function, $F = (F_1, \dots, F_N)$ is a given, and $u = (u_1, \dots, u_N)$ is an unknown real vector-functions, $n \geqslant 2$, $N \geqslant 2$, $\Box := \frac{\partial^2}{\partial t^2} - \Delta$, $\Delta := \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$. We assume that function G together with its first order partial derivatives $\frac{\partial G}{\partial u_i}$, $i = 1, \dots, N$, is continuous in the space $\mathbb{R}^N$.

Consider the Cauchy characteristic problem on finding in the frustrum of the light cone of the future D_T: $|x| < t < T$, $T = \mathrm{const} > 0$, a solution $u(x,t)$ of the system (1.1) by the boundary condition

$$u|_{S_T} = 0, \tag{1.2}$$

where S_T: $t = |x|$, $t \leqslant T$, is the conic surface, characteristic for the system (1.1). For the case when $T = \infty$ we assume that D_∞: $t > |x|$ and $S_\infty = \partial D_\infty$: $t = |x|$.

A question on the existence and uniqueness of global solution of the Cauchy problem for semi-linear scalar equations of the form (1.1) with initial conditions $u|_{t=0} = u_0$, $\frac{\partial u}{\partial t}|_{t=0} = u_1$ has been considered by many authors (see, e.g. [1–11]), while the Cauchy characteristic problem (1.1), (1.2) in the cone of the future for scalar semi-linear hyperbolic equations has been considered in papers [12–18]. In the linear case, as it is known, this problem is well-posed in the corresponding function spaces [18–24].

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Below we give certain conditions for function G providing global solvability of the problem (1.1), (1.2) and a uniqueness of its solution. The uniqueness of this problem for the scalar case has not been considered in papers [12–18].

The paper is organized in the following way. In Section 2 we define a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T and give its a priori estimate. In Section 3 we prove the existence of the strong generalized solution for the problem (1.1), (1.2) of the class W_2^1 in D_T, and in Section 4 we prove the uniqueness of this solution. In Section 4 we give also a proof for the existence of unique global solution of the problem of the class W_2^1 in the cone of the future D_∞: $t > |x|$.

2. A priori estimate of the solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T

Let $\overset{\circ}{W}{}_2^1(D_T, S_T) := \{u \in W_2^1(D_T)\colon\ u|_{S_T} = 0\}$, where $W_2^k(\Omega)$ is the Sobolev space consisting of the elements of $L_2(\Omega)$ having up to k-th order generalized derivatives from $L_2(\Omega)$, inclusively. The equality $u|_{S_T} = 0$ must be understood in the sense of the trace theory [25, p. 71]. Note, that the system (1.1) can be rewritten in the form of one vector equation

$$L_\lambda u := \Box u + \lambda \nabla_u G(u) = F(x,t). \tag{2.1}$$

Definition 2.1. Let $F = (F_1, \ldots, F_N) \in L_2(D_T)$. We call the vector-function $u = u(u_1, \ldots, u_N) \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T if there exists a sequence of vector-functions $u^m \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T) := \{u \in C^2(\overline{D}_T)\colon\ u|_{S_T} = 0\}$ such that $u^m \to u$ in the space $\overset{\circ}{W}{}_2^1(D_T, S_T)$ and $L_\lambda u^m \to F$ in the space $L_2(D_T)$, where operator L_λ is defined by (2.1).

It is obvious, that the classical solution $u \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T)$ of the problem (1.1), (1.2) is a strong generalized solution of the class W_2^1 in the domain D_T.

Under belonging of the vector $v = (v_1, \ldots, v_N)$ to some space X we mean belonging of each component v_i, $1 \leqslant i \leqslant N$, of this vector to the same space X.

Definition 2.2. Let $F \in L_{2,\mathrm{loc}}(D_\infty)$ and $F|_{D_T} \in L_2(D_T)$ for any $T > 0$. Vector-function $u = (u_1, \ldots, u_N) \in \overset{\circ}{W}{}_{2,\mathrm{loc}}^1(D_\infty, S_\infty)$ is called a global strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the cone of the future D_∞, if $u|_{D_T}$ belongs to the space $\overset{\circ}{W}{}_2^1(D_T, S_T)$ and represents a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T in the sense of Definition 2.1 for any $T > 0$.

For the function G from Eq. (1.1) consider the following conditions

$$G(0, \ldots, 0) = 0, \qquad G(u_1, \ldots, u_N) \geqslant -M_1 \sum_{i=1}^N u_i^2 - M_2, \quad M_i = \mathrm{const} \geqslant 0,\ i = 1, 2. \tag{2.2}$$

Lemma 2.1. *Let condition* (2.2) *be fulfilled, $\lambda > 0$ and $F \in L_2(D_T)$. Then for any strong generalized solution $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ of the problem* (1.1), (1.2) *of the class W_2^1 in the domain D_T the following estimate*

$$\|u\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} \leqslant c_1 \|F\|_{L_2(D_T)} + c_2 \tag{2.3}$$

is valid with constants $c_1 = c_1(\lambda, T, M_1) > 0$ and $c_2 = c_2(\lambda, T, M_1, M_2) \geqslant 0$, not depending on u and F.

Proof. Let $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ be a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T. Then, in view of Definition 2.1, there exists a sequence of functions $u^m \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T)$ such that

$$\lim_{m\to\infty} \|u^m - u\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} = 0, \qquad \lim_{m\to\infty} \|L_\lambda u^m - F\|_{L_2(D_T)} = 0. \tag{2.4}$$

The function $u^m \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T)$ can be considered as a solution of the following problem

$$L_\lambda u^m = F^m, \tag{2.5}$$

$$u^m\big|_{S_T} = 0. \tag{2.6}$$

Here

$$F^m = L_\lambda u^m. \tag{2.7}$$

Multiplying both parts of the vector equality (2.5) scalarly by the vector $\frac{\partial u^m}{\partial t}$ in the space $\mathbb{R}^N$ and integrating in the domain D_τ, $0<\tau\leqslant T$, due to (2.1) we have

$$\frac{1}{2}\int\limits_{D_\tau}\frac{\partial}{\partial t}\left(\frac{\partial u^m}{\partial t}\right)^2 dx\,dt-\int\limits_{D_\tau}\Delta u^m\frac{\partial u^m}{\partial t}\,dx\,dt+\lambda\int\limits_{D_\tau}\frac{\partial}{\partial t}G\big(u^m\big)\,dx\,dt=\int\limits_{D_\tau}F^m\frac{\partial u^m}{\partial t}\,dx\,dt. \tag{2.8}$$

Here $(\frac{\partial u^m}{\partial t})^2=\sum_{i=1}^N(\frac{\partial u_i^m}{\partial t})^2$, $\Delta u^m\frac{\partial u^m}{\partial t}=\sum_{i=1}^N\Delta u_i^m\frac{\partial u_i^m}{\partial t}$.

Set Ω_τ: $D_T\cap\{t=\tau\}$ and denote by $\nu=(\nu_1,\dots,\nu_n,\nu_0)$ a unit vector of the outer normal to $S\backslash\{(0,\dots,0,0)\}$. Integrating by parts and taking into account the equalities (2.6) with $\nu|_{\Omega_\tau}=(0,\dots,0,1)$, we obtain

$$\int\limits_{D_\tau}\frac{\partial}{\partial t}\left(\frac{\partial u^m}{\partial t}\right)^2 dx\,dt=\int\limits_{\partial D_\tau}\left(\frac{\partial u^m}{\partial t}\right)^2\nu_0\,ds=\int\limits_{\Omega_\tau}\left(\frac{\partial u^m}{\partial t}\right)^2 dx+\int\limits_{S_\tau}\left(\frac{\partial u^m}{\partial t}\right)^2\nu_0\,ds,$$

$$\int\limits_{D_\tau}\frac{\partial^2 u^m}{\partial x_i^2}\frac{\partial u^m}{\partial t}\,dx\,dt=\int\limits_{\partial D_\tau}\frac{\partial u^m}{\partial x_i}\frac{\partial u^m}{\partial t}\nu_i\,ds-\frac{1}{2}\int\limits_{D_\tau}\frac{\partial}{\partial t}\left(\frac{\partial u^m}{\partial x_i}\right)^2 dx\,dt=\int\limits_{\partial D_\tau}\frac{\partial u^m}{\partial x_i}\frac{\partial u^m}{\partial t}\nu_i\,ds-\frac{1}{2}\int\limits_{\partial D_\tau}\left(\frac{\partial u^m}{\partial x_i}\right)^2\nu_0\,ds$$

$$=\int\limits_{\partial D_\tau}\frac{\partial u^m}{\partial x_i}\frac{\partial u^m}{\partial t}\nu_i\,ds--\frac{1}{2}\int\limits_{S_\tau}\left(\frac{\partial u^m}{\partial x_i}\right)^2\nu_0\,ds-\frac{1}{2}\int\limits_{\Omega_\tau}\left(\frac{\partial u^m}{\partial x_i}\right)^2 dx,$$

$$\int\limits_{D_\tau}\frac{\partial}{\partial t}G\big(u^m\big)\,dx\,dt=\int\limits_{\partial D_\tau}G\big(u^m\big)\nu_0\,ds=\int\limits_{\Omega_\tau}G\big(u^m\big)\,dx.$$

Whence in view of (2.8) we have

$$\int\limits_{D_\tau}F^m\frac{\partial u^m}{\partial t}\,dx\,dt=\int\limits_{S_\tau}\frac{1}{2\nu_0}\left[\sum_{i=1}^n\left(\frac{\partial u^m}{\partial x_i}\nu_0-\frac{\partial u^m}{\partial t}\nu_t\right)^2+\left(\frac{\partial u^m}{\partial t}\right)^2\left(\nu_0^2-\sum_{j=1}^n\nu_j^2\right)\right]ds$$
$$+\frac{1}{2}\int\limits_{\Omega_\tau}\left[\left(\frac{\partial u^m}{\partial t}\right)^2+\sum_{i=1}^n\left(\frac{\partial u^m}{\partial x_i}\right)^2\right]dx+\lambda\int\limits_{\Omega_\tau}G\big(u^m\big)\,dx. \tag{2.9}$$

Since S_τ represents a characteristic surface, then

$$\left(\nu_0^2-\sum_{j=1}^n\nu_j^2\right)\Bigg|_{S_\tau}=0. \tag{2.10}$$

Taking into account that $(\nu_0\frac{\partial}{\partial x_i}-\nu_i\frac{\partial}{\partial t})$, $i=1,\dots,n$, is an inner differential operator on S_τ, then due to (2.6) we have

$$\left(\frac{\partial u^m}{\partial x_i}\nu_0-\frac{\partial u^m}{\partial t}\nu_i\right)\Bigg|_{S_\tau}=0,\quad i=1,\dots,n. \tag{2.11}$$

From (2.9), in view of (2.10) and (2.11), we have

$$\int\limits_{\Omega_\tau}\left[\left(\frac{\partial u^m}{\partial t}\right)^2+\sum_{i=1}^n\left(\frac{\partial u^m}{\partial x_i}\right)^2\right]dx+2\lambda\int\limits_{\Omega_\tau}G\big(u^m\big)\,dx=2\int\limits_{D_\tau}F^m\frac{\partial u^m}{\partial t}\,dx\,dt. \tag{2.12}$$

Since $\lambda>0$ and $2F^m\frac{\partial u^m}{\partial t}\leqslant(\frac{\partial u^m}{\partial t})^2+(F^m)^2$, then in view of (2.12) we have

$$\int\limits_{\Omega_\tau}\left[\left(\frac{\partial u^m}{\partial t}\right)^2+\sum_{i=1}^n\left(\frac{\partial u^m}{\partial x_i}\right)^2\right]dx\leqslant 2\lambda\int\limits_{D_\tau}\big[M_1\big(u^m\big)^2+M_2\big]\,dx\,dt+\int\limits_{D_\tau}\left(\frac{\partial u^m}{\partial t}\right)^2 dx\,dt+\int\limits_{D_\tau}\big(F^m\big)^2\,dx\,dt. \tag{2.13}$$

Since $u^m|_{S_T}=0$, then

$$u^m(x,\tau)=\int\limits_{|x|}^{\tau}\frac{\partial u^m(x,s)}{\partial t}\,ds,\quad (x,\tau)\in\Omega_\tau,\ \tau\leqslant T,$$

whence, using the Schwartz inequality, we obtain

$$\int_{\Omega_\tau} (u^m)^2\,dx \leqslant \int_{\Omega_\tau} \left(\int_{|x|}^{\tau} \frac{\partial u^m(x,s)}{\partial t} \right)^2 dx \leqslant \int_{\Omega_\tau} \left(\tau - |x|\right) \left[\int_{|x|}^{\tau} \left(\frac{\partial u^m(x,s)}{\partial t} \right)^2 ds \right] dx$$
$$\leqslant T \int_{\Omega_\tau} \left[\int_{|x|}^{\tau} \left(\frac{\partial u^m(x,s)}{\partial t} \right)^2 ds \right] dx = T \int_{D_\tau} \left(\frac{\partial u^m}{\partial t} \right)^2 dx\,dt. \tag{2.14}$$

From (2.13) and (2.14) it follows

$$\int_{\Omega_\tau} \left[(u^m)^2 + \left(\frac{\partial u^m}{\partial t} \right)^2 + \sum_{i=1}^{n} \left(\frac{\partial u^m}{\partial x_i} \right)^2 \right] dx \leqslant (2\lambda M_1 + T + 1) \int_{D_\tau} \left[(u^m)^2 + \left(\frac{\partial u^m}{\partial t} \right)^2 + \sum_{i=1}^{n} \left(\frac{\partial u^m}{\partial x_i} \right)^2 \right] dx\,dt$$
$$+ \int_{D_\tau} \left(F^m\right)^2 dx\,dt + 2\lambda M_2\, mes\, D_\tau, \quad \tau \leqslant T. \tag{2.15}$$

Let

$$w(\tau) := \int_{\Omega_\tau} \left[(u^m)^2 + \left(\frac{\partial u^m}{\partial t} \right)^2 + \sum_{i=1}^{n} \left(\frac{\partial u^m}{\partial x_i} \right)^2 \right] dx. \tag{2.16}$$

Since $mes\, D_\tau = \frac{\omega_n}{n+1} \tau^{n+1}$, where ω_n is the volume of the unit ball in the $\mathbb{R}^n$, then in view of (2.15) and (2.16) we have

$$w(\tau) \leqslant (2\lambda M_1 + T + 1) \int_0^{\tau} w(s)\,ds + \left\| F^m \right\|^2_{L_2(D_T)} + \frac{2\lambda M_2 \omega_n}{n+1} T^{n+1}, \quad \tau \leqslant T,$$

whence from the Gronwall's Lemma [26, p. 13] it follows that

$$w(\tau) \leqslant \left(\left\| F^m \right\|^2_{L_2(D_T)} + \frac{2\lambda M_2 \omega_n}{n+1} T^{n+1} \right) \exp(2\lambda M_1 + T + 1)\tau, \quad \tau \leqslant T. \tag{2.17}$$

By help of (2.16) and (2.17) we find that

$$\left\| u^m \right\|^2_{\overset{\circ}{W}{}^1_2(D_T,S_T)} = \int_0^T w(\tau)\,d\tau \leqslant T \left(\left\| F^m \right\|^2_{L_2(D_T)} + \frac{2\lambda M_2 \omega_n}{n+1} T^{n+1} \right) \exp(2\lambda M_1 + T + 1)T. \tag{2.18}$$

From (2.18) we receive

$$\left\| u^m \right\|_{\overset{\circ}{W}{}^1_2(D_T,S_T)} \leqslant c_1 \left\| F^m \right\|_{L_2(D_T)} + c_2. \tag{2.19}$$

Here

$$c_1 = T^{\frac{1}{2}} \exp \frac{1}{2}(2\lambda M_1 + T + 1)T, \qquad c_2 = \left[\frac{2\lambda M_2 \omega_n}{n+1} T^{n+2} \exp(2\lambda M_1 + T + 1)T \right]^{\frac{1}{2}}. \tag{2.20}$$

Due to (2.4) and (2.5), from (2.19) by passing to the limit for $m \to \infty$ we receive (2.3). The lemma is proved. □

3. Solvability of the problem (1.1), (1.2) in the domain D_T

Together with (2.2) consider the following condition for the function G

$$\left| \nabla_u G(u) \right| \leqslant M_3 + M_4 |u|^{\alpha}, \quad \alpha = \text{const} \geqslant 0,\ M_i = \text{const} \geqslant 0,\ i = 3, 4;\ u \in \mathbb{R}^N. \tag{3.1}$$

Remark 3.1. The embedding operator $I : \overset{\circ}{W}{}^1_2(D_T, S_T) \to L_q(D_T)$ represents a linear continuous compact operator for $1 < q < \frac{2(n+1)}{n-1}$, when $n > 1$ [25, p. 86]. At the same time the Nemitski operator $K : [L_q(D_T)]^N \to [L_2(D_T)]^N$, acting by the formula $(Ku)_i = \frac{\partial}{\partial u_i} G(u)$, $i = 1, \dots, N$, $u_j \in L_q(D_T)$, $j = 1, \dots, N$, where function G satisfies the condition (3.1) is continuous and bounded for $q \geqslant 2\alpha$ [27, p. 349], [28, p. 67]. Thus, if $\alpha < \frac{n+1}{n-1}$, i.e. $2\alpha < \frac{2(n+1)}{n-1}$, then there exists number q such that $1 < q < \frac{2(n+1)}{n-1}$ and $q > 2\alpha$. Therefore, in this case the operator

$$K_0 = KI : \left[\overset{\circ}{W}{}_2^1(D_T, S_T)\right]^N \to \left[L_2(D_T)\right]^N \tag{3.2}$$

will be continuous and compact. It is clear that from $u = (u_1, \dots, u_N) \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ it follows that $K_0 u \in L_2(D_T)$ and, if $u^m \to u$ in the space $\overset{\circ}{W}{}_2^1(D_T, S_T)$, then $K_0 u^m \to K_0 u$ in the space $L_2(D_T)$.

Remark 3.2. Before passing to a question of the solvability of the problem (1.1), (1.2) let us consider the same question for the linear case of the required form, when in (1.1) (or in (2.1)) the parameter $\lambda = 0$, i.e. for the problem

$$\begin{aligned} &L_0 u := \Box u = F(x,t), \quad (x,t) \in D_T, \\ &u(x,t) = 0, \quad (x,t) \in S_T. \end{aligned} \tag{3.3}$$

In this case for $F \in L_2(D_T)$ we introduce a notion of a strong generalized solution $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ of the problem (3.3) of the class W_2^1 in the domain D_T for which there exists a sequence of the functions $u^m \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T)$ such that

$$\lim_{m\to\infty} \left\|u^m - u\right\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} = 0, \qquad \lim_{m\to\infty} \left\|L_0 u^m - F\right\|_{L_2(D_T)} = 0.$$

Note that, as it is clear from the proof of Lemma 2.1, for the strong generalized solution $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ of the problem (3.3) of the class W_2^1 in the domain D_T a priori estimate (2.3) takes the following form

$$\|u\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} \leqslant c\|F\|_{L_2(D_T)}, \quad c = T^{\frac{1}{2}} \exp \frac{1}{2}(T+1)T. \tag{3.4}$$

Since the space $C_0^\infty(D_T)$ of finite infinitely differentiable in the D_T functions is dense in the $L_2(D_T)$, then for given $F = (F_1, \dots, F_N) \in L_2(D_T)$ there exists the sequence of vector-functions $F^m = (F_1^m, \dots, F_N^m) \in C_0^\infty(D_T)$ such that $\lim_{m\to\infty} \|F^m - F\|_{L_2(D_T)} = 0$. For fixed m, extending F^m with zero beyond the domain D_T and leaving the same notation for it, we shall have $F^m \in C^\infty(\mathbb{R}_+^{n+1})$, for which the support $\operatorname{supp} F^m \subset D_\infty$, where $\mathbb{R}_+^{n+1} := \mathbb{R}^{n+1} \cap \{t \geqslant 0\}$. Denote by u^m the solution of the Cauchy problem: $L_0 u^m = F^m$, $u^m|_{t=0} = 0$, $\frac{\partial u^m}{\partial t}|_{t=0} = 0$; as it is well known [29, p. 192] the solution exists, it is unique and belongs to the space C^∞. Since $\operatorname{supp} F^m \subset D_\infty$, $u^m|_{t=0} = 0$ and $\frac{\partial u^m}{\partial t}|_{t=0} = 0$, then taking into account geometry of the domain of dependence of the solution of linear wave equation $L_0 u^m = F^m$, we shall have $\operatorname{supp} u^m \subset D_\infty$ [29, p. 191]. Leaving the same notation for the restriction of function u^m in the domain D_T, it is obvious that $u^m \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T)$ and according to Remark 3.2 and (3.4) we shall have

$$\left\|u^m - u^k\right\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} \leqslant c\left\|F^m - F^k\right\|_{L_2(D_T)}. \tag{3.5}$$

Since the sequence $\{F^m\}$ is fundamental in $L_2(D_T)$, then in view of (3.5) the sequence $\{u^m\}$ is fundamental in the complete space $\overset{\circ}{W}{}_2^1(D_T, S_T)$ too. Therefore, there exists a vector-function $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ such that

$$\lim_{m\to\infty} \left\|u^m - u\right\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} = 0$$

and since $L_0 u^m = F^m \to F$ in the space $L_2(D_T)$, then this vector-function due to Remark 3.2 is a strong generalized solution of the problem (3.3) of the class W_2^1 in the domain D_T. The uniqueness of this solution of the space $\overset{\circ}{W}{}_2^1(D_T, S_T)$ follows from a priori estimate (3.4). Therefore, for the solution u of the problem (3.3) we have $u = L_0^{-1} F$, where $L_0^{-1} : [L_2(D_T)]^N \to [\overset{\circ}{W}{}_2^1(D_T, S_T)]^N$ is a linear continuous operator with a norm admitting in view of (3.4) the following estimate

$$\left\|L_0^{-1}\right\|_{[L_2(D_T)]^N \to [\overset{\circ}{W}{}_2^1(D_T,S_T)]^N} \leqslant T^{\frac{1}{2}} \exp \frac{1}{2}(T+1)T. \tag{3.6}$$

Remark 3.3. Let the condition (3.1) be fulfilled and $F \in L_2(D_T)$, $0 \leqslant \alpha < \frac{n+1}{n-1}$. In view of (3.6) and Remark 3.1 it is easy to see that the vector-function $u = (u_1, \dots, u_N) \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ is a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T if and only if u is a solution of the following functional equation

$$u = L_0^{-1}\left(-\lambda \nabla_u G(u) + F\right) \tag{3.7}$$

in the space $\overset{\circ}{W}{}_2^1(D_T, S_T)$.

Rewrite Eq. (3.7) in the form

$$u = Au := L_0^{-1}(-\lambda K_0 u + F), \tag{3.8}$$

where the operator $K_0 : [\overset{\circ}{W}{}_2^1(D_T, S_T)]^N \to [L_2(D_T)]^N$ from (3.2) due to Remark 3.1 is a continuous and compact operator. Therefore, according to (3.6) and (3.8) the operator $A : [\overset{\circ}{W}{}_2^1(D_T, S_T)]^N \to [\overset{\circ}{W}{}_2^1(D_T, S_T)]^N$ is also continuous and compact. At the same time according to Lemma 2.1 for any parameter $\tau \in [0, 1]$ and any solution of the equation $u = \tau Au$ with parameter τ it is valid a priori estimate (2.3) with the same constants c_1 and c_2 from (2.20), not depending on u, F and τ. Therefore, due to the Leray–Schauder Theorem [30, p. 375] Eq. (3.8) has at least one solution $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$, and in view of Remark 3.3 it represents a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T.

Therefore it is valid the following theorem.

Theorem 3.1. *Let $\lambda > 0$, $0 \leqslant \alpha < \frac{n+1}{n-1}$ and the conditions* (2.2), (3.1) *be fulfilled. Then for any $F \in L_2(D_T)$ the problem* (1.1), (1.2) *has at least one strong generalized solution $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ of the class W_2^1 in the domain D_T in the sense of Definition* 2.1.

Remark 3.4. As it follows from the results of the works [12–18], concerning the Cauchy characteristic problem in the light cone of the future for the scalar nonlinear equation $\Box u + \lambda f(u) = F$, violation of the conditions of Theorem 3.1 may cause an absence of a solution of the problem (1.1), (1.2) of the class W_2^1 in the domain D_T in the sense of Definition 2.1.

For $N = 2$ let us consider one class of functions $G(u)$ satisfying the conditions of Theorem 3.1:

$$G(u_1, u_2) = \sum_{i=1}^{m} b_i |u_1|^{\alpha_i} |u_2|^{\beta_i} + \sum_{j=1}^{2} d_j |u_j|^{\gamma_j} + \sum_{i,j=1}^{2} a_{ij} u_i u_j + \sum_{i=1}^{2} a_i u_i,$$

where $\alpha_i, \beta_i > 1$; $(\alpha_i - 1)p_i$, $\beta_i q_i$, $\alpha_i \acute{p}_i$, $(\beta_i - 1)\acute{q}_i < \frac{n+1}{n-1}$; $\frac{1}{p_i} + \frac{1}{q_i} = 1$, $\frac{1}{\acute{p}_i} + \frac{1}{\acute{q}_i} = 1$; $p_i, q_i, \acute{p}_i, \acute{q}_i > 1$; $1 < \gamma_j < \frac{2n}{n-1}$; $b_i, d_j = \text{const} \geqslant 0$; $a_{ij}, a_i = \text{const}$.

4. Uniqueness of the solution and global solvability of the problem (1.1), (1.2)

Let us impose on the function G of Eq. (1.1) additional requirements

$$G \in C^2(\mathbb{R}^N), \quad \left| \frac{\partial^2}{\partial u_i \partial u_j} G(u) \right| \leqslant a + b|u|^{\gamma}, \quad 1 \leqslant i, j \leqslant N; \ \gamma = \text{const} \geqslant 0, \tag{4.1}$$

where $|u| = \|u\|_{\mathbb{R}^N} = (\sum_{i=1}^N u_i^2)^{\frac{1}{2}}$; $a, b = \text{const} \geqslant 0$.

It is obvious that the condition (3.1) follows from (4.1) for $\alpha = \gamma + 1$, and in the case $\gamma < \frac{2}{n-1}$ we shall have $\alpha = \gamma + 1 < \frac{n+1}{n-1}$.

Theorem 4.1. *Let $0 \leqslant \gamma < \frac{2}{n-1}$ and the condition* (4.1) *be fulfilled. Then the problem* (1.1), (1.2) *cannot have more than one strong generalized solution of the class W_2^1 in the domain D_T in the sense of Definition* 2.1.

Proof. Assume that $F \in L_2(D_T)$ and the problem (1.1), (1.2) has two strong generalized solutions u^1 and u^2 of the class W_2^1 in the domain D_T in the sense of Definition 2.1, i.e. there exist two sequences $u^{im} \in \overset{\circ}{C}{}^2(\overline{D}_T, S_T)$, $i = 1, 2$; $m = 1, 2, \ldots,$ of vector-functions such that

$$\lim_{m\to\infty} \|u^{im} - u^i\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} = 0, \qquad \lim_{m\to\infty} \|L_\lambda u^{im} - F\|_{L_2(D_T)} = 0, \quad i = 1, 2. \tag{4.2}$$

Let

$$w = u^2 - u^1, \qquad w^m = u^{2m} - u^{1m}, \qquad F^m = L_\lambda u^{2m} - L_\lambda u^{1m}. \tag{4.3}$$

In view of (4.2) and (4.3) we have

$$\lim_{m\to\infty} \|w^m - w\|_{\overset{\circ}{W}{}_2^1(D_T,S_T)} = 0, \qquad \lim_{m\to\infty} \|F^m\|_{L_2(D_T)} = 0. \tag{4.4}$$

In accordance with (2.1) and (4.3) consider the function $w^m \in \overset{\circ}{C}{}^2(D_T, S_T)$ as a solution of the following problem

$$\Box w^m = -\lambda\left[\nabla_u G\left(u^{2m}\right) - \nabla_u G\left(u^{1m}\right)\right] + F^m, \tag{4.5}$$

$$w^m\big|_{S_T} = 0. \tag{4.6}$$

Multiplying both parts of the vector equation (4.5) scalarly by the vector $\frac{\partial w^m}{\partial t}$ in the space $\mathbb{R}^N$, integrating it by parts in the domain D_T, $0 < \tau \leqslant T$, and taking into account (4.6), in the same way as in the case of receiving the equality (2.12), we shall have

$$\int_{\Omega_\tau}\left[\left(\frac{\partial w^m}{\partial t}\right)^2+\sum_{i=1}^n\left(\frac{\partial w^m}{\partial x_i}\right)^2\right]dx=2\int_{D_\tau}F^m\frac{\partial w^m}{\partial t}\,dx\,dt-2\lambda\int_{D_\tau}\left[\nabla_u G\left(u^{2m}\right)-\nabla_u G\left(u^{1m}\right)\right]\frac{\partial w^m}{\partial t}\,dx\,dt,$$
$$0<\tau\leqslant T. \tag{4.7}$$

Taking into account the equality

$$\frac{\partial}{\partial u_i}G\left(u^{2m}\right)-\frac{\partial}{\partial u_i}G\left(u^{1m}\right)=\sum_{j=1}^N\int_0^1\frac{\partial^2}{\partial u_i\partial u_j}G\left(u^{1m}+s\left(u^{2m}-u^{1m}\right)\right)ds\left(u_j^{2m}-u_j^{1m}\right)$$

we receive

$$\left[\nabla_u G\left(u^{2m}\right)-\nabla_u G\left(u^{1m}\right)\right]\frac{\partial w^m}{\partial t}=\sum_{i,j=1}^N\left[\int_0^1\frac{\partial^2}{\partial u_i\partial u_j}G\left(u^{1m}+s\left(u^{2m}-u^{1m}\right)\right)ds\right]\left(u_j^{2m}-u_j^{1m}\right)\frac{\partial w_i^m}{\partial t}. \tag{4.8}$$

From (4.1) and obvious inequality $|d_1+d_2|^\gamma\leqslant 2^\gamma\max(|d_1|^\gamma,|d_2|^\gamma)\leqslant 2^\gamma(|d_1|^\gamma+|d_2|^\gamma)$ for $\gamma\geqslant 0$, $d_i\in\mathbb{R}$, we have

$$\begin{aligned}\left|\int_0^1\frac{\partial^2}{\partial u_i\partial u_j}G\left(u^{1m}+s\left(u^{2m}-u^{1m}\right)\right)ds\right|&\leqslant\int_0^1\left[a+b\left|(1-s)u^{1m}+su^{2m}\right|^\gamma\right]ds\\&\leqslant a+2^\gamma b\left(\left|u^{1m}\right|^\gamma+\left|u^{2m}\right|^\gamma\right).\end{aligned} \tag{4.9}$$

From (4.8) and (4.9), taking into account (4.3), it follows

$$\begin{aligned}&\left|\left[\nabla_u G\left(u^{2m}\right)-\nabla_u G\left(u^{1m}\right)\right]\frac{\partial w^m}{\partial t}\right|\\&\quad\leqslant\sum_{i,j=1}^N\left[a+2^\gamma b\left(\left|u^{1m}\right|^\gamma+\left|u^{2m}\right|^\gamma\right)\right]\left|w_j^m\right|\left|\frac{\partial w_i^m}{\partial t}\right|\\&\quad\leqslant N^{\frac12}\left[a+2^\gamma b\left(\left|u^{1m}\right|^\gamma+\left|u^{2m}\right|^\gamma\right)\right]\left|w^m\right|\left|\frac{\partial w^m}{\partial t}\right|\\&\quad\leqslant\frac12 N^{\frac12}a\left[\left(w^m\right)^2+\left(\frac{\partial w^m}{\partial t}\right)^2\right]+2^\gamma N^{\frac12}b\left(\left|u^{1m}\right|^\gamma+\left|u^{2m}\right|^\gamma\right)\left|w^m\right|\left|\frac{\partial w^m}{\partial t}\right|.\end{aligned} \tag{4.10}$$

Due to (4.10) from (4.7) we have

$$\begin{aligned}\int_{\Omega_\tau}\left[\left(\frac{\partial w^m}{\partial t}\right)^2+\sum_{i=1}^n\left(\frac{\partial w^m}{\partial x_i}\right)^2\right]dx&\leqslant\int_{D_\tau}\left[\left(\frac{\partial w^m}{\partial t}\right)^2+\left(F^m\right)^2\right]dx\,dt+|\lambda|N^{\frac12}a\int_{D_\tau}\left[\left(w^m\right)^2+\left(\frac{\partial w^m}{\partial t}\right)^2\right]dx\,dt\\&\quad+2^{\gamma+1}|\lambda|N^{\frac12}b\int_{D_T}\left(\left|u^{1m}\right|^\gamma+\left|u^{2m}\right|^\gamma\right)\left|w^m\right|\left|\frac{\partial w^m}{\partial t}\right|dx\,dt.\end{aligned} \tag{4.11}$$

The latter integral in the right side of (4.11) can be estimated by Holder's inequality

$$\int_{D_T}\left(\left|u^{1m}\right|^\gamma+\left|u^{2m}\right|^\gamma\right)\left|w^m\right|\left|\frac{\partial w^m}{\partial t}\right|dx\,dt\leqslant\left(\left\|\left|u^{1m}\right|^\gamma\right\|_{L_{n+1}(D_T)}+\left\|\left|u^{2m}\right|^\gamma\right\|_{L_{n+1}(D_T)}\right)\left\|w^m\right\|_{L_p(D_T)}\left\|\frac{\partial w^m}{\partial t}\right\|_{L_2(D_T)}. \tag{4.12}$$

Here $\frac{1}{n+1}+\frac1p+\frac12=1$, i.e.

$$p=\frac{2(n+1)}{n-1}. \tag{4.13}$$

Since $\dim D_T=n+1$, then according to the Sobolev embedding theorem [25, p. 111] for $1\leqslant q\leqslant\frac{2(n+1)}{n-1}$ we have

$$\|v\|_{L_q(D_T)}\leqslant C_q\|v\|_{W_2^1(D_T)},\quad\forall v\in W_2^1(D_T) \tag{4.14}$$

with positive constant C_q, not depending on $v\in W_2^1(D_T)$.

According to the theorem $\gamma < \frac{2}{n-1}$ and, therefore, $\gamma(n+1) < \frac{2(n+1)}{n-1}$. Thus, from (4.13), (4.14) we receive

$$\big\| |u^{im}|^\gamma \big\|_{L_{n+1}(D_T)} = \|u^{im}\|^\gamma_{L_{\gamma(n+1)}(D_T)} \leqslant C^\gamma_{\gamma(n+1)} \|u^{im}\|^\gamma_{W_2^1(D_T)}, \quad i=1,2;\ m \geqslant 1, \tag{4.15}$$

$$\|w^m\|_{L_p(D_T)} \leqslant C_p \|w^m\|_{W_2^1(D_T)}, \quad m \geqslant 1. \tag{4.16}$$

In view of the first equality from (4.2) there exists natural number m_0 such that for $m \geqslant m_0$ we shall have

$$\|u^{im}\|^\gamma_{W_2^1(D_T)} = \|u^{im}\|^\gamma_{\overset{\circ}{W}{}_2^1(D_T,S_T)} \leqslant \|u^i\|^\gamma_{\overset{\circ}{W}{}_2^1(D_T,S_T)} + 1, \quad i=1,2;\ m \geqslant 1. \tag{4.17}$$

From (4.12), (4.15)–(4.17) it follows that

$$\begin{aligned}
&2^{\gamma+1}|\lambda|N^{\frac{1}{2}} b \int\limits_{D_T} \big(|u^{1m}|^\gamma + |u^{2m}|^\gamma\big)|w^m| \left|\frac{\partial w^m}{\partial t}\right| dx\,dt \\
&\quad \leqslant 2^{\gamma+1}|\lambda|N^{\frac{1}{2}} b C^\gamma_{\gamma(n+1)} \big(\|u^1\|^\gamma_{\overset{\circ}{W}{}_2^1(D_T,S_T)} + \|u^2\|^\gamma_{\overset{\circ}{W}{}_2^1(D_T,S_T)} + 2\big) C_p \|w^m\|_{W_2^1(D_T)} \left\|\frac{\partial w^m}{\partial t}\right\|_{L_2(D_T)} \\
&\quad \leqslant M_3 \left(\|w^m\|^2_{W_2^1(D_T)} + \left\|\frac{\partial w^m}{\partial t}\right\|^2_{L_2(D_T)} \right) \leqslant 2M_3 \|w^m\|^2_{W_2^1(D_T)} \\
&\quad = 2M_3 \int\limits_{D_T} \left[(w^m)^2 + \left(\frac{\partial w^m}{\partial t}\right)^2 + \sum_{i=1}^n \left(\frac{\partial w^m}{\partial x_i}\right)^2 \right] dx\,dt,
\end{aligned} \tag{4.18}$$

where $2M_3 = 2^{\gamma+1}|\lambda|N^{\frac{1}{2}} b C^\gamma_{\gamma(n+1)} (\|u^1\|^\gamma_{\overset{\circ}{W}{}_2^1(D_T,S_T)} + \|u^2\|^\gamma_{\overset{\circ}{W}{}_2^1(D_T,S_T)} + 2) C_p$.

Due to (4.18) from (4.11) we have

$$\begin{aligned}
&\int\limits_{\Omega_\tau} \left[\left(\frac{\partial w^m}{\partial t}\right)^2 + \sum_{i=1}^n \left(\frac{\partial w^m}{\partial x_i}\right)^2 \right] dx \leqslant M_4 \int\limits_{D_T} \left[(w^m)^2 + \left(\frac{\partial w^m}{\partial t}\right)^2 + \sum_{i=1}^n \left(\frac{\partial w^m}{\partial x_i}\right)^2 \right] dx\,dt + \int\limits_{D_T} (F^m)^2\, dx\,dt, \\
&\quad 0 < \tau < T,
\end{aligned} \tag{4.19}$$

where $M_4 = 1 + a|\lambda|N^{\frac{1}{2}} + 2M_3$.

Note, that the inequality (2.14) is valid for w^m too, and, therefore,

$$\int\limits_{\Omega_\tau} (w^m)^2\, dx \leqslant T \int\limits_{D_T} \left(\frac{\partial w^m}{\partial t}\right)^2 dx\,dt \leqslant T \int\limits_{D_T} \left[(w^m)^2 + \left(\frac{\partial w^m}{\partial t}\right)^2 + \sum_{i=1}^n \left(\frac{\partial w^m}{\partial x_i}\right)^2 \right] dx\,dt. \tag{4.20}$$

Putting

$$\xi_m(\tau) := \int\limits_{\Omega_\tau} \left[(w^m)^2 + \left(\frac{\partial w^m}{\partial t}\right)^2 + \sum_{i=1}^n \left(\frac{\partial w^m}{\partial x_i}\right)^2 \right] dx \tag{4.21}$$

and adding (4.19) to (4.20), we receive

$$\xi_m(\tau) \leqslant (M_4 + T) \int\limits_0^\tau \xi_m(s)\, ds + \|F^m\|^2_{L_2(D_T)}.$$

Whence, by the Gronwall Lemma [26, p. 13], it follows that

$$\xi_m(\tau) \leqslant \|F^m\|^2_{L_2(D_T)} \exp(M_4 + T)\tau. \tag{4.22}$$

From (4.21) and (4.22) we have

$$\|w^m\|^2_{W_2^1(D_T)} = \int\limits_0^T \xi_m(\tau)\, d\tau \leqslant T \|F^m\|^2_{L_2(D_T)} \exp(M_4 + T)T. \tag{4.23}$$

In view of (4.3), (4.4) from (4.23) it follows that

$$\|w\|_{W_2^1(D_T)} = \lim_{m\to\infty} \|w - w^m + w^m\|_{W_2^1(D_T)} \leqslant \lim_{m\to\infty} \|w - w^m\|_{W_2^1(D_T)} + \lim_{m\to\infty} \|w^m\|_{W_2^1(D_T)}$$
$$= \lim_{m\to\infty} \|w - w^m\|_{W_2^1(D_T)} = \lim_{m\to\infty} \|w - w^m\|_{\overset{\circ}{W}{}_2^1(D_T, S_T)} = 0.$$

Therefore $w = u_2 - u_1 = 0$, i.e. $u_2 = u_1$. The theorem is proved. □

The next theorem of existence and uniqueness immediately follows from Theorems 3.1 and 4.1.

Theorem 4.2. *Let $\lambda > 0$, $0 \leqslant \gamma < \frac{2}{n-1}$ and the conditions* (2.2), (4.1) *be fulfilled. Then for any $F \in L_2(D_T)$ the problem* (1.1), (1.2) *has unique strong generalized solution $u \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ of the class W_2^1 in the domain D_T in the sense of Definition* 2.1.

The following theorem on existence of global solution of this problem follows from Theorem 4.2.

Theorem 4.3. *Let $\lambda > 0$, $0 \leqslant \gamma < \frac{2}{n-1}$ and the conditions* (2.2), (4.1) *be fulfilled. Then for any $F \in L_{2,\mathrm{loc}}(D_\infty)$ such that $F|_{D_T} \in L_2(D_T)$ for each $T > 0$, the problem* (1.1), (1.2) *has unique global strong generalized solution $u \in \overset{\circ}{W}{}_{2,\mathrm{loc}}^1(D_\infty, S_\infty)$ of the class W_2^1 in the cone of the future D_∞ in the sense of Definition* 2.2.

Proof. According to Theorem 4.2 when the conditions of Theorem 4.3 are fulfilled for $T = k$ there exists unique strong generalized solution $u_k \in \overset{\circ}{W}{}_2^1(D_T, S_T)$ of the problem (1.1), (1.2) of the class W_2^1 in the domain $D_{T=k}$ in the sense of Definition 2.1. Since $u_{k+1}|_{D_{T=k}}$ is also a strong generalized solution of the problem (1.1), (1.2) of the class W_2^1 in the domain $D_{T=k}$, then in view of Theorem 4.2 we have $u_k = u_{k+1}|_{D_{T=k}}$. Thus one can construct unique global generalized solution $u \in \overset{\circ}{W}{}_{2,\mathrm{loc}}^1(D_\infty, S_\infty)$ of the problem (1.1), (1.2) of the class W_2^1 in the cone of the future D_∞ in the sense of Definition 2.2 in the following way:

$$u(x,t) = u_k(x,t), \quad (x,t) \in D_\infty,\ k = [t] + 1,$$

where $[t]$ is an integer part of the number t. □

References

[1] K. Jörgens, Das Anfangswertproblem im Grossen fur eine Klasse nichtlinearer Wellengleichungen, Math. Z. 77 (1961) 295–308.
[2] L.E. Segal, The global Cauchy problem for a relativistic scalar field with power interaction, Bull. Soc. Math. France 91 (1963) 129–135.
[3] H.A. Levine, Instability and nonexistence of global solutions to nonlinear wave equations of the form $Pu_{tt} = -Au + F(u)$, Trans. Amer. Math. Soc. 192 (1974) 1–21.
[4] F. John, Blow-up of solutions of nonlinear wave equations in three space dimensions, Manuscripta Math. 28 (1–3) (1979) 235–268.
[5] T. Kato, Blow-up of solutions of some nonlinear hyperbolic equations, Comm. Pure Appl. Math. 33 (4) (1980) 501–505.
[6] T.C. Sideris, Nonexistence of global solutions to semilinear wave equations in high dimensions, J. Differential Equations 52 (3) (1984) 378–406.
[7] V. Georgiev, H. Lindblad, C.D. Sogge, Weighted Strichartz estimates and global existence for semilinear wave equations, Amer. J. Math. 119 (6) (1997) 1291–1319.
[8] E. Mitidieri, S.I. Pohozaev, A priori estimates and the absence of solutions of nonlinear partial differential equations and inequalities, Tr. Mat. Inst. Steklova 234 (2001) 1–384 (in Russian); English transl.: Proc. Steklov Inst. Math. 234 (3) (2001) 1–362.
[9] R. Ikehata, K. Tanizawa, Global existence of solutions for semilinear damped wave equations in R^n with noncompactly supported initial data, Nonlinear Anal. 61 (2005) 1189–1208.
[10] Y. Zhou, Global existence and nonexistence for a nonlinear wave equation with damping and source terms, Math. Nachr. 278 (11) (2008) 1341–1358.
[11] C.E. Kenig, F. Merle, Global well-posedness, scattering and blow-up for the energy-critical focusing non-linear wave equation, Acta Math. 201 (2008) 147–212.
[12] S. Kharibegashvili, Some multidimensional problems for hyperbolic partial differential equations and systems, Mem. Differential Equations Math. Phys. 37 (2006) 1–136.
[13] S. Kharibegashvili, On the existence or the absence of global solutions of the Cauchy characteristic problem for some nonlinear hyperbolic equations, Bound. Value Probl. 2005 (3) (2005) 359–376.
[14] S. Kharibegashvili, On the nonexistence of global solutions of the characteristic Cauchy problem for a nonlinear wave equation in a conical domain, Differ. Uravn. 42 (2) (2006) 261–271 (in Russian); English transl.: Differ. Equ. 42 (2) (2006) 279–290.
[15] S. Kharibegashvili, On the global solvability of the Cauchy characteristic problem for a nonlinear wave equation in a light cone of the future, Mem. Differential Equations Math. Phys. 42 (2007) 49–68.
[16] S. Kharibegashvili, On the Solvability of the Cauchy characteristic problem for a nonlinear equation with iterated wave operator in the principal part, J. Math. Anal. Appl. 338 (1) (2008) 71–81.
[17] S. Kharibegashvili, On the solvability of the Cauchy characteristic problem for some nonlinear wave equations in a light cone of the future, Differ. Uravn. 44 (1) (2008) 129–139 (in Russian); English transl.: Differ. Equ. 44 (1) (2008) 135–146.
[18] S. Kharibegashvili, Boundary value problems for some classes of nonlinear wave equations, Mem. Differential Equations Math. Phys. 48 (2009) 1–114.
[19] M. Reisz, L'intégrale de Riemann–Liouville et le problème de Cauchy, Acta Math. 81 (1949) 1–223.
[20] J. Hadamard, Lectures on Cauchy's Problem in Partial Differential Equations, Yale University Press, New Haven, 1923.
[21] R. Courant, Partial Differential Equations, New York/London, 1962.
[22] F. Cagnac, Problème de Cauchy sur un conoïde caractéristique, Ann. Mat. Pura Appl. (4) 104 (1975) 355–393.
[23] L.E. Lundberg, The Klein–Gordon equation with light-cone data, Comm. Math. Phys. 62 (2) (1978) 107–118.

[24] A.V. Bitsadze, Some Classes of Partial Differential Equations, Nauka, Moscow, 1981 (in Russian).
[25] O.A. Ladyzhenskaya, Boundary Value Problems of Mathematical Physics, Nauka, Moscow, 1973 (in Russian).
[26] D. Henry, Geometrical Theory of Semi-Linear Parabolic Equations, Mir, Moscow, 1985 (translated into Russian); English original: Lecture Notes in Math., vol. 840, Springer-Verlag, Berlin/New York, 1981.
[27] M.A. Krasnosel'skiĭ, P.P. Zabreĭko, E.I. Pustyl'nik, P.E. Sobolevskiĭ, Integral Operators in Spaces of Summable Functions, Nauka, Moscow, 1966 (in Russian).
[28] A. Kufner, S. Futchik, Nonlinear Differential Equations, Nauka, Moscow, 1988 (translated into Russian); English original: Stud. Appl. Mech., vol. 2, Elsevier Scientific Publishing Co., Amsterdam/New York, 1980.
[29] L. Hörmander, Linear Partial Differential Operators, Mir, Moscow, 1965 (translated into Russian); English original: Grundlehren Math. Wiss., vol. 116, Academic Press/Springer-Verlag, New York/Berlin/Göttingen/Heidelberg, 1963.
[30] V.A. Trenogin, Functional Analysis, Nauka, Moscow, 1993 (in Russian).