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GAUGE INVARIANT QUANTIZATION OF $\mathbf{AdS_3 \times S^3}$ PARTICLE DYNAMICS

Introduction

Quantization of particle dynamics in AdS backgrounds is an important step towards the computation of string energy spectrum [1, 2], that plays a major role in the study of the AdS/CFT correspondence (for a review see [3]).

The present paper is a continuation of our previous work [4], where we investigated particle type string solutions in $\mathrm{AdS}_3 \times \mathrm{S}^3$ by the Pohlmeyer reduction. Quantization of these solutions was done in [5], using the orbit method in the conformal gauge.

In this paper we apply a gauge invariant approach to the $\mathrm{AdS}_3 \times \mathrm{S}^3$ particle dynamics and derive a canonical structure on the physical phase space. We construct a complete set of gauge invariant variables with the help of the isometry group dynamical integrals. They provide a nine dimensional manifold of the mass-shell and the tenth gauge invariant variable is obtained from the analysis of the symplectic structure. The additional coordinate becomes an angle variable, which is canonically conjugated to the angular momentum on S^3. The obtained canonical structure leads to the Hollstein-Primakoff representation for the isometry group generators.

Setting up Notation

Let us denote coordinates of $\mathbb{R}^{2,2}$ and $\mathbb{R}^4$ by $(X^{0'}, X^0,\ X^1, X^2)$ and (Y^1, Y^2, Y^3, Y^4), respectively. The AdS_3 and S^3 spaces are defined by the embedding conditions

$$X{\cdot}X \equiv -X_{0'}^2 - X_0^2 + X_1^2 + X_2^2 = -1\,, \qquad Y{\cdot}Y \equiv Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 = 1\,. \quad (1)$$

Introducing the matrices

$$g = \begin{pmatrix} X^{0'} + X^2 & X^1 + X^0 \\ X^1 - X^0 & X^{0'} - X^2 \end{pmatrix},\ h = \begin{pmatrix} Y^4 + iY^3 & Y^2 + iY^1 \\ -Y^2 + iY^1 & Y^4 - iY^3 \end{pmatrix}, \quad (2)$$

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one finds that the conditions (1) are equivalent to $g \in \mathrm{SL}(2,\mathbb{R})$ and $h \in \mathrm{SU}(2)$. Thus, AdS_3 and S^3 are identified with the group manifolds $\mathrm{SL}(2,\mathbb{R})$ and $\mathrm{SU}(2)$, respectively.

We choose the following basis in the $\mathfrak{sl}(2,\mathbb{R})$ algebra

$$\mathbf{t}_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \ \mathbf{t}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \ \mathbf{t}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{3}$$

The matrices $\mathbf{t}_\mu$ ($\mu = 0, 1, 2$) satisfy the relation

$$\mathbf{t}_\mu \, \mathbf{t}_\nu = \eta_{\mu\nu} \, \mathbf{I} + \epsilon_{\mu\nu}{}^{\rho} \, \mathbf{t}_\rho \ , \tag{4}$$

where $\mathbf{I}$ is the unit matrix, $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1)$ and $\epsilon_{\mu\nu\rho}$ is the Levi-Civita tensor, with $\epsilon_{012} = 1$. The inner product defined by $\langle \, \mathbf{t}_\mu \, \mathbf{t}_\nu \, \rangle = \frac{1}{2} \, \mathrm{tr}(\mathbf{t}_\mu \, \mathbf{t}_\nu)$ provides the isometry between $\mathfrak{sl}(2,\mathbb{R})$ and 3d Minkowski space.

A similar basis in the $\mathfrak{su}(2)$ algebra is given by $\mathbf{s}_n = i\boldsymbol{\sigma}_n$ ($n = 1, 2, 3$), where $\boldsymbol{\sigma}_n$ are the Pauli matrices ($\boldsymbol{\sigma}_1 = \mathbf{t}_1 \ \boldsymbol{\sigma}_2 = -i\mathbf{t}_0, \ \boldsymbol{\sigma}_3 = \mathbf{t}_2$), and they form the algebra

$$\mathbf{s}_m \, \mathbf{s}_n = -\delta_{mn} \, \mathbf{I} - \epsilon_{mnl} \, \mathbf{s}_l \ . \tag{5}$$

Hence, $\mathfrak{su}(2)$ is isometric to $\mathbb{R}^3$ by the inner product $\langle \mathbf{s}_m \, \mathbf{s}_n \rangle \equiv -\frac{1}{2} \, \mathrm{tr}(\mathbf{s}_m \, \mathbf{s}_n) = \delta_{mn}$.

From (4) follow the identities

$$\mathbf{ab} + \mathbf{ba} = 2\langle \mathbf{ab} \rangle \mathbf{I} \ , \qquad \mathbf{aba} = 2\langle \mathbf{ab} \rangle \mathbf{a} - \langle \mathbf{a}^2 \rangle \mathbf{b} \ , \tag{6}$$

where $\mathbf{a}$ and $\mathbf{b}$ are two arbitrary vectors of $\mathfrak{sl}(2,\mathbb{R})$. In particular, $\mathbf{a}^2 = \langle \mathbf{a}^2 \rangle \mathbf{I}$.

Similarly, if $\mathbf{u}$ and $\mathbf{v}$ are elements of $\mathfrak{su}(2)$, from (5) we find $\mathbf{u}^2 = -\langle \mathbf{u}^2 \rangle \mathbf{I}$ and

$$\mathbf{uv} + \mathbf{vu} = -2\langle \mathbf{uv} \rangle \mathbf{I} \ , \qquad \mathbf{uvu} = -2\langle \mathbf{uv} \rangle \mathbf{u} + \langle \mathbf{u}^2 \rangle \mathbf{v} \ . \tag{7}$$

The matrices g and h in (2) and their inverse group elements can be written as

$$g = X^{0'} \, \mathbf{I} + X^\mu \, \mathbf{t}_\mu \ , \qquad h = X_4 \, \mathbf{I} + X_n \, \mathbf{s}_n \ , \tag{8}$$

$$g^{-1} = X^{0'} \, \mathbf{I} - X^\mu \, \mathbf{t}_\mu \ , \qquad h^{-1} = Y_4 \, \mathbf{I} - Y_n \, \mathbf{s}_n \ , \tag{9}$$

and by (4) and (5) one finds the following relations between the length elements

$$\langle \, (g^{-1} \, dg) \, (g^{-1} \, dg) \rangle = dX \cdot dX \ , \ \langle \, (h^{-1} \, dh) \, (h^{-1} \, dh) \rangle = dY \cdot dY \ . \tag{10}$$

Particle Dynamics in $\mathrm{AdS}_3 \times \mathrm{S}_3$

The dynamics of a massive particle in $\mathrm{AdS}_3 \times \mathrm{S}_3$ can be described by the action

$$S = \int \mathrm{d}\tau \left[\frac{\langle g^{-1} \, \dot{g} \, g^{-1} \, \dot{g} \rangle + \langle h^{-1} \, \dot{h} \, h^{-1} \, \dot{h} \rangle}{2\lambda} - \frac{\lambda \mu^2}{2} \right] . \tag{11}$$

Here τ is an evolution parameter, λ plays the role of a Lagrange multiplier and μ is the particle mass. In the first order formalism this action is equivalent to

$$S = \int \mathrm{d}\tau \, \left[\langle R\, g^{-1}\, \dot{g}\rangle + \langle R^s\, h^{-1}\, \dot{h}\rangle - \frac{\lambda}{2}\left(\langle RR\rangle + \langle R^s R^s\rangle + \mu^2\right)\right] \ , \tag{12}$$

where R and R^s are Lie algebra valued ($R \in \mathfrak{sl}(2,\mathbb{R}),\ R^s \in \mathfrak{su}(2)$) phase space variables. The Hamilton equations obtained from (12) read

$$g^{-1}\, \dot{g} = \lambda R\ ,\ \dot{R} = 0\ , \qquad h^{-1}\, \dot{h} = \lambda R^s\ ,\ \dot{R}^s = 0\ , \tag{13}$$

and the variation of (12) with respect to λ provides the mass-shell condition

$$\langle\, R\, R\,\rangle + \langle\, R^s\, R^s\,\rangle + \mu^2 = 0\ . \tag{14}$$

To prepare the system for quantization one has to find physical variables on the constraint surface (14) and calculate the reduction of the symplectic form defined by (12). This can be done either by gauge fixing or in a gauge invariant way, using a complete set of gauge invariant variables. Here we follow the gauge invariant approach.

In order to find a complete set of gauge invariant variables we introduce the 'left' Lie algebra valued quantities

$$L = g\, R\, g^{-1}\ , \qquad L^s = h\, R^s\, h^{-1}\ , \tag{15}$$

which have the same norm as the 'right' ones

Since the worldlines are timelike, L and R have to be time-like elements of $\mathfrak{sl}(2,\mathbb{R})$, i.e. $\langle\, L\, L\,\rangle = -\mu_a^2 = \langle\, R\, R\,\rangle$, with $\mu_a > 0$. We also introduce the norm of L^s and R^s by $\langle\, L^s\, L^s\,\rangle = \mu_s^2 = \langle\, R^s\, R^s\,\rangle$, and write the mass-shell condition (14) as $\mu_a^2 = \mu_s^2 + \mu^2$.

The first order action (12) defines the pre-symplectic form of the system

$$\theta = \langle R g^{-1} \mathrm{d}g\rangle + \langle R^s h^{-1} \mathrm{d}h\rangle\ , \tag{16}$$

which leads to the following Poisson brackets

$$\begin{aligned}
&\{L_\mu,\, L_\nu\} = -2\epsilon_{\mu\nu}{}^{\rho}\, L_\rho\ , \quad \{R_\mu, R_\nu\} = 2\epsilon_{\mu\nu}{}^{\rho}\, R_\rho\ , \qquad \{L_\mu,\, R_\nu\} = 0\ ,\\
&\{L^s_m,\, L^s_n\} = 2\epsilon_{mnl}\, L^s_l\ , \qquad \{R^s_m,\, R^s_n\} = -2\epsilon_{mnl}\, R^s_l\ , \quad \{L^s_m,\, R^s_n\} = 0\ .
\end{aligned} \tag{17}$$

where $L_\mu = \langle\, \mathbf{t}_\mu\, L\,\rangle,\ R_\mu = \langle\, \mathbf{t}_\mu\, R\,\rangle,\ L^s_m = \langle\, \mathbf{s}_m\, L^s\,\rangle,\ R^s_m = \langle\, \mathbf{s}_m\, R^s\,\rangle$. From these Poisson brackets follow that the components $L_\mu,\ R_\mu,\ L^s_m,\ R^s_m$ have vanishing Poisson brackets with the constraint (14). Hence, the components are gauge invariant and, therefore, their Poisson brackets algebra (17) will be preserved after the reduction to the physical phase space. By (13), these components are time independent as well. Obviously, they are the isometry Noether charges of (11) and satisfy the relations

$$L_\mu L^\mu = -\mu_a^2 = R_\mu R^\mu\ , \qquad L^s_m L^s_m = \mu_s^2 = R^s_m R^s_m\ . \tag{18}$$

Taking into account (14), we conclude that the number of independent components is nine. On the other hand, the number of independent physical variables is ten. The missing physical variable has to be constructed from the group elements g and h.

Note that a given L and R define the group element $g \in \mathrm{SL}(2,\mathbb{R})$ up to a rotation angle [6]. A similar statement is also valid for the SU(2) part. To extract these angle parameters, we introduce normalized Lie algebra elements

$$\hat{l} = L/\mu_a \ , \quad \hat{r} = R/\mu_a \ ; \qquad \hat{l}^s = L^s/\mu_s \ , \quad \hat{r}^s = R^s/\mu_s \tag{19}$$

and define auxiliary variables for the $\mathrm{SL}(2,\mathbb{R})$ part

$$\cosh 2\alpha_{_L} = -\langle\, \hat{l}\, \mathbf{t}_0 \,\rangle, \qquad \cosh 2\alpha_{_R} = -\langle\, \hat{r}\, \mathbf{t}_0 \,\rangle \ , \tag{20}$$

$$\hat{n}_{_L} = \frac{[\mathbf{t}_0,\, \hat{l}]}{2\sinh 2\alpha_{_L}} \ , \qquad \hat{n}_{_R} = -\frac{[\mathbf{t}_0,\, \hat{r}]}{2\sinh 2\alpha_{_R}} \ . \tag{21}$$

Similar variables for the SU(2) part are given by

$$\cos 2\alpha^s_{_L} = \langle\, \hat{l}^s\, \mathbf{s}_3 \,\rangle, \qquad \cos 2\alpha^s_{_R} = \langle\, \hat{r}^s\, \mathbf{s}_3 \,\rangle \ , \tag{22}$$

$$\hat{n}^s_{_L} = \frac{[\mathbf{s}_3,\, \hat{l}^s]}{2\sin 2\alpha^s_{_L}} \ , \qquad \hat{n}^s_{_R} = -\frac{[\mathbf{s}_3,\, \hat{r}^s]}{2\sin 2\alpha^s_{_R}} \ , \tag{23}$$

and we use the relations

$$e^{\alpha\, \hat{n}} = \cosh\alpha\, \mathbf{I} + \sinh\alpha\ \hat{n} \ , \qquad e^{\alpha_s\, \hat{n}_s} = \cos\alpha_s\, \mathbf{I} + \sin\alpha_s\ \hat{n}_s \ , \tag{24}$$

which hold for unit and spacelike $\hat{n} \in \mathfrak{sl}(2,\mathbb{R})$ and unit $\hat{n}_s \in \mathfrak{su}(2)$.

From (6), (7) and (19)-(24) follows that

$$e^{-\alpha_L\, \hat{n}_L}\, \hat{l}\, e^{\alpha_L\, \hat{n}_L} = \mathbf{t}_0 \ , \qquad e^{\alpha_R\, \hat{n}_R}\, \hat{r}\, e^{-\alpha_R\, \hat{n}_R} = \mathbf{t}_0 \ , \tag{25}$$

$$e^{-\alpha^s_L\, \hat{n}^s_L}\, \hat{l}^s\, e^{\alpha^s_L\, \hat{n}^s_L} = \mathbf{s}_3 \ , \qquad e^{\alpha^s_R\, \hat{n}^s_R}\, \hat{r}^s\, e^{-\alpha^s_R\, \hat{n}^s_R} = \mathbf{s}_3 \ . \tag{26}$$

Applying these equations to (15), we find the group elements

$$g = e^{\alpha_L\, \hat{n}_L}\, e^{-\varphi_a\, \mathbf{t}_0}\, e^{\alpha_R\, \hat{n}_R} \ , \qquad h = e^{\alpha^s_L\, \hat{n}^s_L}\, e^{\varphi_s\, \mathbf{s}_3}\, e^{\alpha^s_R\, \hat{n}^s_R} \ , \tag{27}$$

where φ_a and φ_s are the angle parameters mentioned above.

The insertion of (27) into (16) leads to the following pre-symplectic form

$$\theta = \mu_a \mathrm{d}\varphi_a + \theta_{_L} + \theta_{_R} + \mu_s \mathrm{d}\varphi_s + \theta^s_{_L} + \theta^s_{_R} \ , \quad \text{with} \tag{28}$$

$$\begin{aligned} \theta_{_L} &= \frac{\mu_a\langle\, [\hat{l},\, \mathbf{t}_0]\, \mathrm{d}\hat{l} \,\rangle}{4(1 - \langle\, \mathbf{t}_0\, \hat{l} \,\rangle)} = H_{_L}\, \mathrm{d}\phi_{_L} \ , \qquad \theta_{_R} = \frac{\mu_a\langle\, [\mathbf{t}_0,\, \hat{r}]\, \mathrm{d}\hat{r} \,\rangle}{4(1 - \langle\, \mathbf{t}_0\, \hat{r} \,\rangle)} = H_{_R} \mathrm{d}\phi_{_R} \ , \\ \theta^s_{_L} &= \frac{\mu_s\langle\, [\hat{l}^s,\, \mathbf{s}_3]\, \mathrm{d}\hat{l}^s \,\rangle}{4(1 + \langle\, \mathbf{s}_3\, \hat{l}^s \,\rangle)} = H^s_{_L} \mathrm{d}\phi^s_{_L} \ , \quad \theta^s_{_R} = \frac{\mu_s\langle\, [\mathbf{s}_3,\, \hat{r}^s]\, \mathrm{d}\hat{r}^s \,\rangle}{4(1 + \langle\, \mathbf{s}_3\, \hat{r}^s \,\rangle)} = H^s_{_R} \mathrm{d}\phi^s_{_R} \ . \end{aligned} \tag{29}$$

$$H_L = \frac{\mu_a}{2}(\hat{l}^0 - 1)\,,\ \tan\phi_L = \frac{\hat{l}_1}{\hat{l}_2}\,,\quad H_R = \frac{\mu_a}{2}(\hat{r}^0 - 1),\ \tan\phi_R = \frac{\hat{r}_2}{\hat{r}_1}\,,$$
$$H^s_L = \frac{\mu_s}{2}(1 - \hat{l}^s_3)\,,\ \tan\phi^s_L = \frac{\hat{l}^s_2}{\hat{l}^s_1}\,,\quad H^s_R = \frac{\mu_s}{2}(1 - \hat{r}^s_3)\,,\ \tan\phi^s_R = \frac{\hat{r}^s_1}{\hat{r}^s_2}\,. \tag{30}$$

The differential of (28) then takes a canonical form

$$\omega = \mathrm{d}\mu_a \wedge \mathrm{d}\varphi_a + \mathrm{d}H_L \wedge \mathrm{d}\phi_L + \mathrm{d}H_R \wedge \mathrm{d}\phi_R + $$
$$+\mathrm{d}\mu_s \wedge \mathrm{d}\varphi_s + \mathrm{d}H^s_L \wedge \mathrm{d}\phi^s_L + \mathrm{d}H^s_R \wedge \mathrm{d}\phi^s_R\,, \tag{31}$$

and we obtain the dynamical integrals in terms of canonical variables

$$L^0 = \mu_a + 2H_L\,,\qquad R^0 = \mu_a + 2H_R\,,$$
$$L_\pm = \sqrt{\mu_a H_L + H_L^2}\,e^{\pm i\phi_L}\,,\qquad R_\pm = \sqrt{\mu_a H_R + H_R^2}\,e^{\pm i\phi_R}\,, \tag{32}$$

$$L^s_3 = \mu_s - 2H^s_L\,,\qquad R^s_3 = \mu_s - 2H^s_R\,,$$
$$L^s_\pm = \sqrt{\mu_s H^s_L - H^{s\,2}_L}\;e^{\pm i\phi^s_L}\,,\qquad R_\pm = \sqrt{\mu_s H^s_R - H^{s\,2}_R}\;e^{\pm i\phi^s_R}\,, \tag{33}$$

where $L_\pm = \frac{1}{2}(L_2 \pm iL_1)$, $R_\pm = \frac{1}{2}(R_1 \pm iR_2)$, $L^s_\pm = \frac{1}{2}(L^s_1 \pm iL^s_2)$ and $R^s_\pm = \frac{1}{2}(R^s_2 \pm iR^s_1)$, $\mu_a = \sqrt{\mu^2 + \mu_s^2}$ and one can read of μ_a's canonical conjugated variable from (31).

Quantization

The form (32)-(33) dictates the realization of the symmetry generators in terms of creation-annihilation operators, known as the Holstein-Primakoff transformation

$$L^0 = \mu_a + 2a^\dagger_L a_L\,,\qquad R^0 = \mu_a + 2a^\dagger_R a_R\,,$$
$$L_+ = a^\dagger_L\sqrt{\mu_a + a^\dagger_L a_L}\,,\qquad R_+ = a^\dagger_R\sqrt{\mu_a + a^\dagger_R a_R}\,, \tag{34}$$
$$L_- = \sqrt{\mu_a + a^\dagger_L a_L}\;a_L\,,\qquad R_- = \sqrt{\mu_a + a^\dagger_R a_R}\;a_R\,,$$

$$L^s_3 = \mu_s - 2a^{s\,\dagger}_L a^s_L\,,\qquad R^s_3 = \mu_s - 2a^{s\,\dagger}_R a^s_R\,,$$
$$L^s_+ = a^{s\,\dagger}_L\sqrt{\mu_s - a^{s\,\dagger}_L a^s_L}\,,\qquad R^s_+ = a^{s\,\dagger}_R\sqrt{\mu_s - a^{s\,\dagger}_R a^s_R}\,, \tag{35}$$
$$L^s_- = \sqrt{\mu_s - a^{s\,\dagger}_L a^s_L}\;a^s_L\,,\qquad R^s_- = \sqrt{\mu_s - a^{s\,\dagger}_R a^s_R}\;a^s_R\,.$$

This yields a representation of $\mathfrak{sl}(2,\mathbb{R})_L \oplus \mathfrak{sl}(2,\mathbb{R})_R \otimes \mathfrak{su}(2)_L \otimes \mathfrak{su}(2)_R$, with the basis vectors $|\,\mu_a;\,n_L\,\rangle\,|\,\mu_a;\,n_R\,\rangle\,|\,\mu_s;\,n^s_L\,\rangle\,|\,\mu_a;\,n^s_R\,\rangle$ where n_L, n_R, μ_s, n^s_L, n^s_R are nonnegative integers, with $n^s_L \le \mu_s$ and $n^s_R \le \mu_s$.

The representation is characterized by the Casimir numbers identified with

$$C_A = -L_\mu L^\mu = -R_\mu R^\mu = \mu_a(\mu_a - 2)\,,$$
$$C_S = L^s_m L^s_m = R^s_m R^s_m = \mu_s(\mu_s + 2)\,. \tag{36}$$

According to (14), $C_A = C_S + \mu^2$, which provides $\mu_a = 1 + \sqrt{\mu^2 + (\mu_s + 1)^2}$.

The spectrum of the energy operator $E = \frac{1}{2}\left(L^0 + R^0\right)$ then reads

$$E = \mu_a + m_{_L} + m_{_R} \ . \tag{37}$$

where μ_a is the lowest energy level for a given spherical orbital momentum μ_s.

In this way we reproduce the result obtained in [5] and [7].

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