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ON MEAN-VALUE PROPERTY FOR HYPERBOLIC EQUATIONS

The mean-value property for equations with real characteristics which to some extent differs from the same property for harmonic functions or, in general, for elliptic equations, is given. For some hyperbolic equations, this property is expressed in terms of relations between values of each of their solutions at opposite apexes of an arbitrary characteristic quadrangle. To the class of such behavior belong, for example, hyperbolic equations with general integrals representable explicitly in the form

$$A(x,t,u) = f\big[B(x,t,u)\big] + g\big[B(x,t,u)\big].$$

The arguments B and C of sufficiently smooth arbitrary functions f, g are characteristic combinations determined by the principal part of the equation. This class involves second order equations with coefficients as polynomials of lower derivatives of unknown solutions. For the string equation these combinations are

$$A(x,t,u) = u, \quad B(x,t,u) = x+t, \quad C(x,t,u) = x-t.$$

As an example, we consider the following hyperbolic equation:

$$L[u] \equiv \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial t^2} + \Big(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\Big)\frac{\partial^2 u}{\partial x\,\partial t} - \frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial x^2} = \frac{1}{x}\frac{\partial u}{\partial t}\Big(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}\Big)$$

with an admissible parabolic degeneracy. Because of the singularity of the lower term in the right-hand side, it can be considered as a nonlinear generalization of the Euler-Darboux equations. Both characteristic families are given by the following relations:

$$u = \text{const}, \quad x - t = \text{const},$$

and the general integral has one of the two equivalent forms

$$u(x,t) = f\big[sg(x-t)\big] \ \text{ or } \ f\big[u(x,t)\big] + xg(x-t) = 0, \ \ \forall\, f, g \in C^2(R^1).$$

Assuming that a set of four points (x_1, t_1), (x_3, t_3) and (x_2, t_2), (x_4, t_4) are the pairwise opposite apexes of the characteristic quadrangle, the equality of products $x_1 x_3 = x_2 x_4$ is proved. Consequently, the mean-value property in the given case is expressed as "proportional argument property". Relying

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on this property, the initial and mixed problems can be considered. There is one of the nonlinear versions of the Goursat characteristic problem and among them is: find a solution of the equation by its values $\varphi(x)$ on $J = \big\{(x,t) : t = x,\ 0 < a \le x \le b\big\}$, if the arc of the first characteristic family is given as

$$\gamma = \big\{(x,t) : t = \mu(x),\ \mu'(x) \neq 1,\ \mu(a) = a,\ a \le x \le k\big\}.$$

A solution of the problem and of its domain of definition is to be found simultaneously. Assuming the existence of a single sufficiently smooth inverse M to the function

$$\nu(x) = \big[\mu(x) - x\big],$$

the existence of a hyperbolic solution to the problem is proved. The solution is defined in the domain

$$D = \Big\{(x,t) : (x-t) \in \big[0, b - \mu(b)\big],\ \frac{x}{M(x-t)} \in [0,1]\Big\}.$$

In such a case, the solution is unique. If these conditions aren't fulfilled, then neither solutions nor domains of their definition are unique.

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