Survey of Buffer Phenomenon

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1. *Buffer phenomenon* is an important example of mutually enriching interaction of theoretical research of a mathematical model for a real event and deep penetration into the essence of this event. Detailed investigation of such a phenomenon allowed introduction of new elements into the interpretation of "nonlinear world" notion.

   Oscillatory objects with distributed parameters are found in different fields of science, new hardware and modern technologies. Dynamics of such objects is simulated by systems of partial differential equations with boundary conditions. Stable cycle corresponding to a self-oscillatory regime is a periodic in time solution.

   Such a *boundary-value problem* contains also the parameters and it is essential to determine the number of coexisting self-oscillatory process for different values of parameters. Hence, it is a purely mathematical problem: studying *the dependence of a number of stable cycles on parameters* in a boundary-value problem.

2. *Buffer phenomenon* in a mathematical model of a distributed oscillatory system is observed when the considered boundary-value problem under proper choice of the values of parameters can contain any finite preliminarily fixed number of different stable cycles. In general case, buffer phenomenon of a parameter-dependent dynamic system has the following property: any *a priori* chosen finite number of single-type attractors exist in the system's phase space when the parameters are chosen properly.

   Obviously, the problem on investigation of time-periodic regimes in oscillatory objects with distributed parameters first was stated by A. A. Vitt [1].

3. Detailed statement of strict mathematical theory of buffer phenomenon can be found in papers and monographs [2–6]. The considered mathematical models are nonlinear boundary-value problems for the systems of partial differential equations of hyperbolic or parabolic type. It is essential that buffer phenomenon itself is specific to bifurcation process, in the course of which unlimited increase of the number of coexisting stable attractors takes place.

   The conducted research showed that buffer phenomenon is "typical" of rather broad class of mathematical models that adequately describe many nonlinear oscillatory processes in natural science (radiophysics [7, 8], mechanics [9], optics [10], combustion theory [11], ecology [12], neurodynamics [13]). Besides, relation of buffer phenomenon to such nontrivial phenomena as turbulence and dynamic chaos has been traced [14–16].

   The study of typical scenarios of accumulation of attractors in different dynamic systems is quite topical. Four scenarios of this kind have been discovered so far: Vitt, Turing, Hamilton, and homoclinic mechanisms of accumulation of attractors.

4. The situation in which *Vitt mechanism* is implemented is typical of a large class of physical processes described by hyperbolic equations. It consists in the following.

   Assume that in the problem of stability of equilibrium zero-state of some hyperbolic system there is a critical case of denumerable number of eigenvalues, and when parameters of the system change, a part of spectrum points is successively displaced to the right complex half-plane. Then in case of no certain resonant correlations between the system's eigenfrequencies, is observed unlimited accumulation of quasiharmonic stable cycles, and each cycle originates from zero-state of equilibrium as an unstable one, and then it acquires stability, rising its amplitude [3, 5, 7, 8].

5. *Turing mechanism*: when parameters change, each individual cycle first gains stability and then loses it once again. Thus, though the total number of attractors grows, their set is constantly renovated. As it is shown in [6], such situation is implemented in reaction-diffusion-type systems under proportional decrease of diffusion coefficients, but it can also show up in the systems with delay under unlimited increase of delay time.

6. As to finite-dimensional systems, the elementary mechanism of buffer onset is *Hamilton scenario* illustrated in [17, 18] by 2D-mappings from mechanics and systems of ordinary differential equations that are close to 2D-Hamiltonian ones. It should be noted that Hamilton mechanism has been the less studied one, though it is illustrated by many examples like pendulum-type equations with time-periodic small additional components [19].

7. In the case of systems of ordinary differential equations there are other, much more complex, mechanisms of accumulation of stable cycles that result from so-called homoclinic contacts existing in such systems; such mechanisms can also be conventionally called *homoclinic.* Among many results obtained for the systems with homoclinic structures, let us comment on three of them [20–22].

8. Note that buffer phenomenon in self-excited oscillators with a section of long two-wire line in a feedback circuit has been experimentally shown to be feasible [2, 8].

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