An Infinite Dimensional Generalization of Weak Exponents for Solutions of Equations in Total Derivatives

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Let E and F be real Banach spaces, $K \subset E$ be a closed convex cone with a bounded base. Consider a linear completely integrable equation in total derivatives

$$y'h = A(x)hy, \;\; y \in F, \;\; h \in E, \;\; x \in E, \tag{1}$$

with a bounded continuous coefficient $A : E \to L(E, L(F, F))$ (here and in the sequel, we use notation and notions from [1]). Let $\mathcal{E}(y)$ be a set of all linear continuous functionals $\mu \in E^*$ such that the inequality $\limsup\limits_{x \to \infty,\, x \in K} \|x\|^{-1}(\ln y(x) + \mu x) \le 0$ holds.

In [2], the interrelation is established between characteristic functionals and (weak) characteristic exponents of solutions of equation (1) for a finite-dimensional E in the form $\mathcal{E}(y) = \mathcal{E}(\exp \psi[y])$, where $\psi[y](x) := \overline{\lim\limits_{t \to +\infty}}\, t^{-1} \ln y(tx)$ is a modified exponent of a solution y. This result is valid only for a finite-dimensional E. Therefore it is necessary to generalize the above notions in order to obtain some analog of the statement in [2] for the settings of infinite-dimensional E.

For this, we introduce new exponent $\psi[y](\varphi)$ by means of the formula

$$\psi[y](\varphi) := \overline{\lim_{t \to +\infty}}\, t^{-1} \ln \big\|y(\varphi(t))\big\|,$$

as the functional on the space Φ of continuous functions $\varphi : [0, +\infty[\to K$ such that $\|\varphi(t)\| \to \infty$, as $t \to +\infty$ and $\sup \|\varphi(t)\|/t < +\infty$.

Theorem. *The inclusion $\lambda \in \mathcal{E}(y)$ holds if and only if for any $\varphi \in \Phi$ the inequality $\psi[y](\varphi) + \lambda(\varphi) \le 0$, where $\lambda(\varphi) := \underline{\lim\limits_{t \to \infty}}\, t^{-1} \lambda\varphi(t)$, holds.*

References

[1] I. V. Gaĭshun, Linear total differential equations. (Russian) *"Nauka i Tekhnika", Minsk*, 1989.

[2] E. K. Makarov, On the interrelation between characteristic functionals and weak characteristic exponents. (Russian) *Differentsial'nye Uravneniya* **30** (1994), No. 3, 393–399; translation in *Differential Equations* **30** (1994), No. 3, 362–367.