On Hybrid Stabilization

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1 Introduction

Hybrid systems are those combining both discrete and continuous dynamics. Many examples of hybrid systems can be found in manufacturing systems, intelligent vehicle highway systems, verious chemical plants. Hybrid systems also arise when there is a necessity of combining logical decision with the generation of continuous control laws.

An important question is how to stabilize a continuous control plant through an interaction with a discrete time controller. Such a "hybrid" feedback may help when the ordinary feedback fails to stabilize the system.

An example of a linear system which cannot be stabilized by the ordinary output feedback is **the harmonic oscillator**:

$$\frac{d\xi}{dt} = \eta, \quad \frac{d\eta}{dt} = -\xi + u, \quad u = u(y), \quad y = \xi. \tag{1.1}$$

The only measured quantity (output), which is allowed to control, is the position variable ξ. Although this last system is both controllable and observable, it cannot be stabilized by (even discontinuous) output feedbacks.

It was however shown by Z. Artstein (1995), there exists a special *hybrid feedback control*, under which system (1.1) becomes asymptotically stable. Z. Artstein conjectured also that hybrid controls can stabilize general linear systems of ordinary differential equations.

2 The Main Result

We give here the following affirmative answer to Artstein's conjecture on the existence of a hybrid stabilizer.

Theorem 2.1. *Under assumptions of controllability of (A, B) and observability of (A, C) the system*

$$\dot{x} = Ax + Bu,$$
$$u = u(y), \quad y = Cx$$

is stabilizable by a hybrid feedback control designed with the help of a discrete automaton which has at most countable number of locations.

Proof is based on the classical stabilization technique as well as on some recent results in the **theory of functional-differential equations** in an essential way.

3 Applications to Theorem 2.1

We give here two examples.

1. Predator-Prey Interactions

An example of a situation, where hybrid feedback controls may be of use, is given by a population model with an arbitrary number of species, some of them being observable and the others not. Although such a model is nonlinear, but linearization about the equilibrium state provides a linear system with a control depending on *a part* of variables while the rest variables may be not observable at all, so that these variables cannot be used for setting up a control function. A stabilization of the unstable equilibrium state may become then problematical if we use ordinary feedback, only. What does help is *hybrid feedback controls*.

2. A Contribution to the Theory of Love Affairs

Here is another example which illustrates the power of stabilization by hybrid feedback controls. The mathematical model for the dynamics of love affairs is given by a 2×2-system of linear equations (Strogatz, 1994). We consider the following particular case, which is called *the star-crossed romance between Romeo and Juliet.*

$$\dot{R} = aJ, \quad \dot{J} = -bR, \tag{3.1}$$

$$R(t) = \text{Romeo's love/hate for Juliet at time } t,$$

$$J(t) = \text{Juliet's love/hate for Romeo at time } t$$

(love gives positive sign to variables, while hate makes variables negative).

From the system (3.1) we obtain the following tragic picture.

The more Romeo loves Juliet, the more Juliet wants to run away and hide. But when Romeo gets discouraged and backs off, Juliet begins to find him strangely attractive. Romeo, on the other hand, tends to echo her; he warms up when she loves him, and grows cold when she hates him... The sad outcome of their affair is, of course, a never-ending cycle of love and hate, because solutions of the system (3.1) *are ellipses with a center at* $(0,0)$.

We observe that ordinary feedback controls do not help Romeo and Juliet in this unpleasant situation. If, for instance, we insert a feedback control like $u = \alpha R$ in the first equation suddenly turns Romeo either into an "eager beaver" ($\alpha > 0$), or into a "cautious lover" ($\alpha < 0$) which seems to be quite unrealistic as soon as one particular Juliet is concerned. The same applies to Juliet. The only possibility is therefore to try making influence on the constants a and b in the system (3.1). But any feedback control like $u = \alpha J$ or/and $v = \beta R$ will never change the sad and tragic picture of never-ending ellipses in the phase-plane, because the corresponding coefficient matrix will always have imaginary eigenvalues!

We propose *a hybrid feedback scenario*, which according to Theorem 2.1 does make solutions of the system (3.1) asymptotically stable. This scenario can be described explicitly and does lead to a kind of "happy end". Naturally, the relationship between Romeo and Juliet will fizzle out to mutual indifference and the disaster will be prevented.

Unfortunately, it is impossible to use a similar procedure to help Romeo and Juliet becoming eventually daring to each other, because hybrid feedback controls can stabilize solutions, but they cannot exclude oscillations.