**G. Berikelashvili**

**A. Razmadze Mathematical Institute, Georgian Academy of Sciences**
**Tbilisi, Georgia**

## ON A NONLOCAL GENERALIZATION OF THE DIRICHLET PROBLEM

Let $\Omega = \{(x_1, x_2) :\ 0 < x_k < 1,\ k = 1,2\}$ be a unit square with a boundary $\Gamma$, and let $\Gamma_1 = \{(0, x_2) :\ 0 < x_2 < 1\}$, $\Gamma_* = \Gamma \backslash \Gamma_1$, $\xi \in (0;1]$, $\varepsilon \in (0;1)$.

Consider the nonlocal boundary value problem

$$\mathcal{L}u = f(x),\ x \in \Omega,\ u(x) = 0, x \in \Gamma_*,\ l(u) = 0, 0 < x_2 < 1, \tag{1}$$

where

$$\mathcal{L}u := \sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\Big(a_{ij}\frac{\partial u}{\partial x_j}\Big) - a_0 u,\ l(u) := \int\limits_0^{\xi} \beta(x)u(x)\,dx_1,\ \beta(x) := \varepsilon x_1^{\varepsilon-1}/\xi^{\varepsilon}$$

and the constant coefficients satisfying the following conditions

$$\sum_{i,j=1}^{2} a_{ij}t_it_j \geq \nu(t_1^2 + t_2^2),\ \nu > 0,\ a_0 \geq 0.$$

By $L_2(\Omega, \rho)$ we denote the weighted Lebesgue space; $\rho(x) := (x_1/\xi)^{\varepsilon}$ for $x_1 < \xi$, $\rho(x) := 1$ for $x_1 \geq \xi$.

We denote by $W_2^1(\Omega, \rho)$ the weighted Sobolev space with the norm

$$\|u\|_{W_2^1(\Omega,\rho)} = \Big(\|u\|^2_{L_2(\Omega,\rho)} + \Big\|\frac{\partial u}{\partial x_1}\Big\|^2_{L_2(\Omega,\rho)} + \Big\|\frac{\partial u}{\partial x_2}\Big\|^2_{L_2(\Omega,\rho)}\Big)^{1/2}.$$

Define the subspace of the space $W_2^1(\Omega, \rho)$ which can be obtained by closing the set

$$\overset{*}{C}{}^{\infty}(\overline{\Omega}) = \Big\{u \in C^{\infty}(\overline{\Omega}) :\ \operatorname{supp} u \cap \Gamma_* = \emptyset,\ l(u) = 0,\ 0 < x_2 < 1\Big\}$$

with the norm $\|\cdot\|_{W_2^1(\Omega,\rho)}$. Denote it by $\overset{*}{W}{}_2^1(\Omega, \rho)$.

Let the right-hand side $f(x)$ in equation (1) be a linear continuous functional on $\overset{*}{W}{}_2^1(\Omega, \rho)$ which can be represented as

$$f = f_0 + \partial f_1/\partial x_1 + \partial f_2/\partial x_2,\quad f_k(x) \in L_2(\Omega, \rho),\quad k = 0, 1, 2.$$

**Theorem 1.** *Problem* (1) *has a unique solution from* $\overset{*}{W}{}_2^1(\Omega, \rho)$.

By passing to the limit $\xi \to 0$ the nonlocal condition $l(u) = 0$ transforms to $u(0, x_2) = 0$, while Theorem 1 to the well-known theorem on the existence and uniqueness of a solution of the Dirichlet problem. In this sense, the nonlocal problem (1) can be regarded as a generalization of the Dirichlet boundary value problem.