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CYLINDRICAL BENDING OF SHELLS WITH CUSPED EDGES

The main purpose of this paper is to study the Initial Boundary Value Problems (IBVPs) for an elastic cusped prismatic shell (see [1], [2]), with the projection $\Omega = \{(x_1, x_2): \ -\infty < x_1 < \infty, \ 0 \leq x_2 \leq \ell\}$.

The equation of the cylindrical bending of a prismatic shell has the form

$$\big(D(x_2) w,_{22}(x_2,t)\big)_{22} = q(x_2,t) - 2\rho h(x_2) \frac{\partial^2 w(x_2,t)}{\partial t^2}, \quad 0 \leq x_2 \leq \ell, \qquad (1)$$

where $w(x_2,t)$ is a deflection of the shell, $q(x_2,t)$ is an intensivity of a load, ρ is a density, $2h(x_2) = h_0 x_2^{\alpha/3}(\ell - x_2)^{\beta/3}$, $h_0, \alpha\,\beta = \mathrm{const} > 0$, is the thickness of the shell, $D(x_2)$ is a flexural-rigidity (see [3]).

In the case under consideration, bending moment and interecting force have the following forms $M_2(x_2,t) := -D(x_2)w,_{22}(x_2,t)$, $Q_2(x_2,t) := M_{2,2}(x_2,t)$.

At the points 0 and ℓ all above quantities are defined as the corresponding limits when $x_2 \to 0_+$ and $x_2 \to \ell_-$. On the cusped edge $x_2 = 0$ $(x_2 = \ell)$ admissible are only four different pairs of the boundary data (see [2]).

Let $w(\cdot,t) \in C^4(]0,\ell[)$, $w(x_2,\cdot) \in C^1(t \geq 0) \cap C^2(t > 0)$, $w(x_2,t) \in C(t \geq 0,\ 0 \leq x_2 \leq \ell)$.

$$\begin{matrix} w(\cdot,t) \\ (w,_2(\cdot,t)) \end{matrix} \left\{ \begin{array}{l} \in C([0,\ell]) \ \text{ when } \ 0 \leq \alpha,\beta < 2, \ \ (0 \leq \alpha,\beta < 1) \\ \left\{ \begin{array}{ll} \in C([0,\ell[), & \text{when } \ 0 \leq \alpha < 2, \ \ \beta \geq 2, \\ = O(1), \ \ x_2 \to \ell_-, & \qquad (0 \leq \alpha < 1, \ \ \beta \geq 1) \end{array} \right. \\ \left\{ \begin{array}{ll} \in C(]0,\ell]), & \text{when } \ \alpha \geq 2, \ \ 0 \leq \beta < 2, \\ = O(1), \ \ x_2 \to 0_+, & \qquad (\alpha \geq 1, \ \ 0 \leq \beta < 1) \end{array} \right. \end{array} \right. \qquad (2)$$

We assume that initial datas are satisfying same boundary conditions, which we have for w, and they are from the class (2).

Equation (1) under usual initial and above mentioned admissible boundary conditions has a unique solution. The IBVPs can be reduced to the integro-differential equations with symmetric kernel, which we solve by the Fourier method. The convergence of appropriate series are proved.

References

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3. S. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells. *Mcgraw-hill Book Company, INC, New York–Toronto–London*, 1959.