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BOUNDARY VALUE PROBLEMS FOR TWO-DIMENSIONAL DELAYED DIFFERENTIAL SYSTEMS

The question on the solvability of the boundary value problem

$$\frac{du_i(t)}{dt} = f_i\big(t, u_1(\tau_1(t)), u_2(\tau_2(t))\big) \quad (i = 1, 2), \tag{1}$$

$$u_1(a) = \alpha u_2(a), \quad \beta_1 u_1(b) + \beta_2 u_2(b) = \gamma \tag{2}$$

is investigated. Here $f_i : [a, b] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ and $\tau_i : [a, b] \to [a, b]$ $(i = 1, 2)$ are continuous functions, $\mathbb{R}_+ = [0, +\infty[$, $\alpha \in \mathbb{R}_+$, $\beta_i \in \mathbb{R}_+$ $(i = 1, 2)$, $\beta_1 + \beta_2 > 0$, $\gamma > 0$, and

$$\tau_i(t) \le t, \quad f_i(t, 0, 0) = 0 \text{ for } a \le t \le b \ \ (i = 1, 2).$$

In particular, the following theorem is proved.

Theorem. *For the solvability of problem* (1), (2) *it is sufficient one of the following five conditions to be fulfilled:*

(i) $\beta_1 > 0$, $\beta_2 > 0$;

(ii) $\beta_1 = 0$, $\beta_2 = 1$, *and there exists a positive constant* ℓ *such that*

$$f_1(t, x, y) \le \ell\big[1 + f_2(t, x, y)\big] \ \text{ for } \ a \le t \le b, \ \ x \ge 0, \ \ 0 \le y \le \gamma;$$

(iii) $\beta_1 = 0$, $\beta_2 = 1$, *and*

$$\tau_1(t) < t \ \text{ for } \ a < t \le b;$$

(iv) $\alpha > 0$, $\beta_1 = 0$, $\beta_2 = 0$, *and there exists a positive constant* ℓ *such that*

$$f_2(t, x, y) \le \ell\big[1 + f_1(t, x, y)\big] \ \text{ for } \ a \le t \le b, \ \ 0 \le x \le \gamma, \ \ y \ge 0;$$

(v) $\alpha > 0$, $\beta_1 = 1$, $\beta_2 = 0$, *and*

$$\tau_2(t) < 0 \ \text{ for } \ a < t \le b.$$